Markets with Frictions: From Set-Valued Risk Measures to Utility Maximization

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Overview: First lecture.

1. Complete vs. incomplete risk preferences for multivariate positions
2. Eligible assets and (partial) liquidation
3. Set-valued risk measures: The regulator case
4. Set-valued risk measures: Market extensions
5. Scalarizations and the preference rectangle

The basic problem is posed by a possible incompleteness of the initial preference given over a set of multivariate positions. The standard theory of utility representation is not applicable anymore, and recent generalizations to multi-utility representations do not answer the question how to '"maximize" such a multi-utility function. The same applies to risk measures and minimization.

We start the first part by discussing simple examples illustrating the problem. Then, a decomposition based on partial liquidations is used to obtain the space of eligible assets. This space provides potential risk compensating portfolios, and its power set serves as an image space for set-valued risk measure. It will easily be realized that properties of the risk measure restrict the power set to much more convenient constructions like the set of all closed or closed and convex sets. Primal and dual representations are obtained with emphasis on the set-valued replacements for the expectation operator which features in dual representations of scalar risk measures. Finally, market models of conical and convex type enter the picture and lead to so-called market extensions of set-valued risk measures. The first part is concluded by a recovery of complete risk preferences, but now a whole family of such preferences corresponding to a single incomplete risk preference.

Overview: Second lecture.

1. Complete vs. incomplete utility preferences for multivariate positions
2. The component-wise approach to utility maximization
3. Set-valued utility maximization: Duality

4. Set-valued certainty equivalents and related topics

5. Conclusions and open problems

The central problem of the second part is the maximization of a vector of utility functions which will be obtained by assuming a few not too restrictive, but still disputable assumptions to the underlying preference. An entirely new approach to utility maximization problems is presented which relies on a recent duality theory for set-valued optimization problems. It is shown that one can obtain the same type of results as in the scalar theory, but involving set-valued functions and interpreting notions like "supremum" and "maximum" in a set-valued sense. A then pretty straightforward development produces set-valued analogs of the certainty equivalent for multivariate positions. We conclude with a discussion of potential paths for future research.