Stochastic mortality under measure changes

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Talk based on joint work with David Blake (Cass), Lorenzo Pitotti (Imperial/Algorithmics), Ariel Sun (Imperial/RMS), Michel Denuit (UC Louvain), Pierre Devolder (UC Louvain)

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AGENDA

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MOTIVATION

Stochastic mortality models

- doubly stochastic/Cox setting ubiquitous
- pricing/valuation approaches vs. realistic risk analysis
- computational tractability vs. empirical evidence/performance

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Same model for different purposes: is it asking too much?

- real world and risk-neutral world
- comparability: capital modelling and market-consistent valuation
- Cox setting has drawbacks, but can survive changes of measures

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Mortality risk premia

- structure of risk premia often restrictive: broader structure valuable, often needed by setting
- reduced-form vs. structural/equilibrium approaches
  - bottom-up approaches to mortality risk premia
  - examples: asymmetric information, funding costs

AGENDA

COX SETTING

m individuals aged $x_1, \ldots, x_m$ at time 0

- τ^i individual i's random time of death
- $N_t = (N_t^1, \ldots, N_t^m)$, $N_t^i = 1_{\tau^i \leq t}$

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Information structure $\mathbb{F} = \mathbb{G} \vee \mathbb{H}$

- $\mathbb{G} = (\mathcal{G}_t)_{t \geq 0}$ carries information about relevant risk factors (health status, reference populations, interest rates, etc.)
- $\mathbb{H} = (\mathcal{H}_t)_{t \geq 0} = \vee_{i=1}^m \mathbb{H}^i$ carries information about death occurrences
  - each $\mathbb{H}^i = (\mathcal{H}_t^i)_{t \geq 0}$ augmented filtration generated by N_t^i

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Cox / doubly stochastic / conditionally Poisson assumption

- Conditional on $\mathcal{G}_\infty$, each N^i coincides with first jump of conditionally Poisson process with $\mathbb{G}$-predictable intensity $(\mu_t^i)_{t \ge 0}$
- Conditional survival probabilities

$$\mathbb{P}(\tau^i > T | \mathcal{F}_t) = 1_{\tau^i > t} E\left[e^{-\int_t^T \mu_s^i \mathrm{d}s} \middle| \mathcal{G}_t \right]$$

PRO'S

- (Semi)explicitly survival probabilities in some settings (e.g., affine)

$$E\left[e^{-\int_0^T \mu_t^i \mathrm{d}t}\right] = e^{A(0;T)+B(0;T)\cdot X_0}, \quad \mu_t^i = g^i(t, X_t)$$

- Spread-based approach to market-consistent pricing and reserving

$$E^{\widetilde{\mathbb{P}}}\left[\int_0^T e^{-\int_0^t r_s \mathrm{d}s} D_{t-} \mathrm{d}N_t^i + e^{-\int_0^T r_s \mathrm{d}s} S_T 1_{\tau^i > T}\right]$$
$$= E^{\widetilde{\mathbb{P}}}\left[\int_0^T e^{-\int_0^t (r_s+\mu_s^i)\mathrm{d}s} D_t \mu_t^i \mathrm{d}t + e^{-\int_0^T (r_t+\mu_t^i)\mathrm{d}t} S_T\right]$$

- Can easily simulate random death times

$$\tau^i = \inf\left\{t > 0 : \int_0^t \mu_s^i > \Theta\right\}, \quad \Theta \sim \mathrm{Exp}(1)$$

CON'S

Information and death/survival probabilities

- Cannot update conditional survival probability based on death occurrences

$$\mathbb{P}(\tau^i > T | \mathcal{G}_t \vee \mathcal{H}_t^1 \vee \ldots \vee \mathcal{H}_t^n) = \mathbb{P}(\tau^i > T | \mathcal{G}_t \vee \mathcal{H}_t^i)$$

- Example: Learning about the underlying force of mortality

Inconsistency across settings

- Cox may hold under $\mathbb{P}$ but not under $\widetilde{\mathbb{P}} \sim \mathbb{P}$
- Think of real-world/risk-neutral ESGs
- Example: risk-neutral Lee-Carter family

Inconsistency with approximate hedging methods

- natural incomplete market approaches inconsistent with Cox under $\widetilde{\mathbb{P}} \sim \mathbb{P}$
- Example: Mean-Variance hedging with bespoke longevity swap

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EQUIVALENT MARTINGALE MEASURES

Girsanov-Meyer theorem

$$\frac{d\widetilde{\mathbb{P}}}{d\mathbb{P}} = \exp\left(-\int_0^T \frac{1}{2}||\eta_s||^2 ds - \int_0^T \eta_s \cdot dW_s\right)$$
$$\prod_{i=1}^m \left(1 + \phi^i_{\tau^i} 1_{\tau^i \leq T}\right) \exp\left(\int_0^{\min(\tau^i, T)} \phi^i_s \mu^i_s ds\right)$$

- $\widetilde{W}_t = W_t + \int_0^t \eta_s ds$ B.m. under $\widetilde{\mathbb{P}}$
- each τ^i has intensity $1_{\tau^i > t}(1 + \phi^i_t)\mu^i_t$ under $\widetilde{\mathbb{P}}$

Cox setting survives when switching to $\widetilde{\mathbb{P}} \sim \mathbb{P}$ if $(\eta, \phi^1, \ldots, \phi^m)$ $\mathbb{G}$-predictable

MORTALITY RISK PREMIA

Systematic mortality risk

- affects the conditional death probability of each individual in the portfolio
- channelled by $(\eta_t)_{t\geq 0}$

Unsystematic mortality risk

- depends on portfolio size, $m - \sum_{i=1}^{m} N_t^i$
- jointly captured by $\phi_t^1(1-N_t^1), \dots, \phi_t^m(1-N_t^m)$

Individual death timing risk

- captured by each $\phi_t^i(1-N_t^i)$
- relevant whenever insurance demand (e.g., pricing) or policyholders' preferences (American-type guarantees, dynamic adverse selection) matter

The last two involve a change in intensity process, $\mu_t^i \rightsquigarrow \mu_t^i(1+\phi_t^i)$, the first one does not.

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EXAMPLE: LEARNING

Death times $\tau^1, \ldots, \tau^m$ have common intenstiy μ

- assume that $Y := \log \mu$ evolves according to

$$\mathrm{d}Y_t = (a(t) + b(t)Y_t + c(t)\psi)\mathrm{d}t + \sigma(t, Y_t)\mathrm{d}W_t$$

Insurer observes Y

- recovers σ from quadratic variation of Y, and draws inferences about drift in Bayesian fashion, backing out from observations of Y the true value of ψ
- think of insurer endowed with $(\mathbb{F}^Y, \widetilde{\mathbb{P}})$, with $\widetilde{\mathbb{P}}$ subjective probability measure reflecting Bayesian updating based on prior beliefs on ψ

$$\eta_t = \frac{c(t)(\psi - \Psi_t)}{\sigma(t, Y_t)} \in \mathcal{G}_t, \quad \phi^i = 0$$

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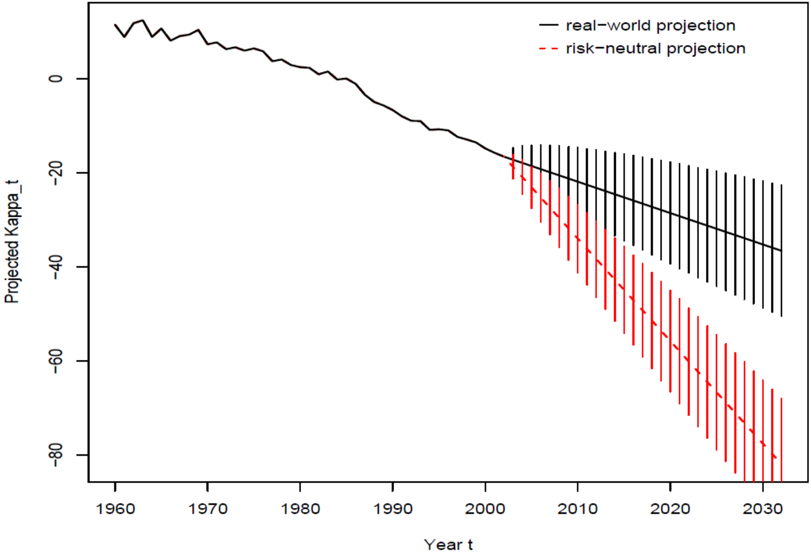
Suppose the insurer uses (Y, N)

- $\eta, \phi^1, \ldots, \phi^m$ depend on N: the Cox setting does not survive...

LEE-CARTER GOES RISK-NEUTRAL



Source: Biffis/Denuit/Devolder (2010)

EXAMPLE: MEAN-VARIANCE HEDGING

Liability

- portfolio of indexed survival benefits (e.g., $\sum_{i=1}^{m} 1_{\tau^i > T} f(S_T)$)

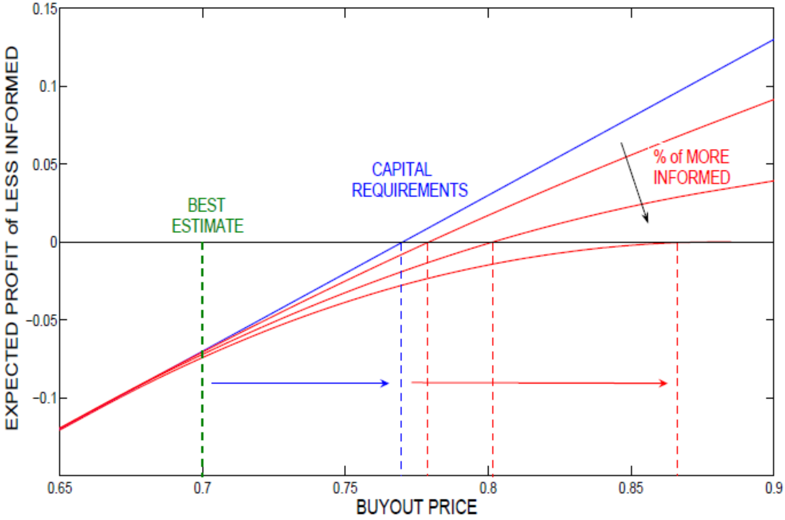
Tradeables

- index $(S_t)_{t \geq 0}$
- bespoke longevity swap, it spans $(N_t)_{t \geq 0}$

Approximate hedging method

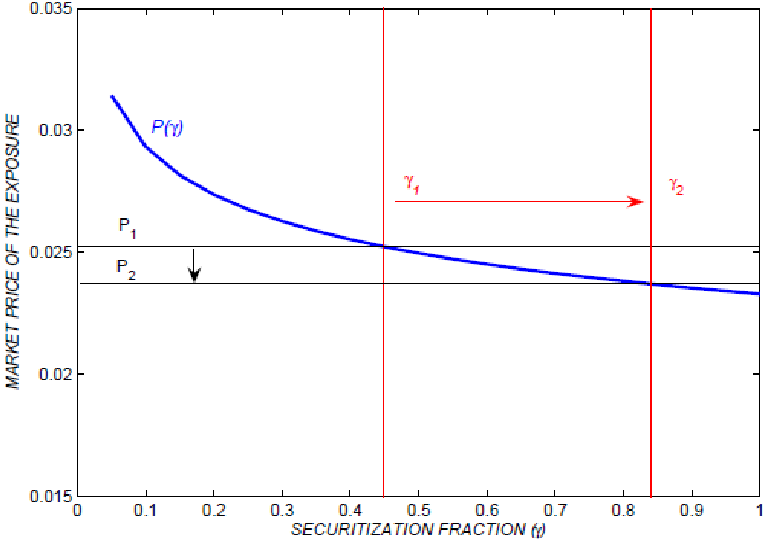
- minimize mean-square error of A/L mismatch at $T > 0$
- $\widetilde{\mathbb{P}}$ entails $\mathbb{F}$-predictable $(\eta, \phi^1, \ldots, \phi^m)$
  - Cox setting does not survive change of measure
- $\widetilde{\mathbb{P}}$ entails $\phi^i \neq 0$
  - portfolio size attracts a risk premium

EXAMPLE: ADVERSE SELECTION IN THE BUYOUT MARKET



Source: Biffis/Blake (2011)

EXAMPLE: SECURITIZATION WITH ASYMMETRIC INFORMATION



Source: Biffis/Blake (2010a)

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SOME TRANSACTIONS

Date	Hedger	Size	Term (yrs)	Type	Interm./supplier
Jan 08	Lucida	Not disclosed	10	indexed	JPM ILS funds
Jul 2008	Canada Life	GBP 500m	40	indemnity	JPM ILS funds
Feb 2009	Abbey Life	GBP 1.5bn	run-off	indemnity	DB ILS funds Partner Re
Mar 2009	Aviva	GBP 475m	10	indemnity	RBS
Jun 2009	Babcock International	GBP 750m	50	indemnity	Credit Suisse Pacific Life Re
Jul 2009	RSA	GBP 1.9bn	run-off	indemnity	GS (Rothesay Life)
Dec 2009	Berkshire Council	GBP 750m	run-off	indemnity	Swiss Re
Feb 2010	BMW UK	GBP 3bn	run-off	indemnity	DB Paternoster
Dec 2010	Swiss Re (Kortis bond)	USD 50m	8	indexed	ILS funds
Feb 2011	Pall Pension Fund	GBP 70m	10	indexed	JPM
...	...	...	...	...	...

LONGEVITY SWAPS

Main issues

- Dodd-Frank, EMIR bespoke solutions
- counterparty risk is bilateral
- longevity risk premium meaningless if MTM/collateral flows ignored

Reference model

- IRS market: bilaterally collateralized, cash collateral in over 90% of the cases
- collateral thresholds based on mark-to-model, mortality experience, credit ratings, CDS spreads, etc.

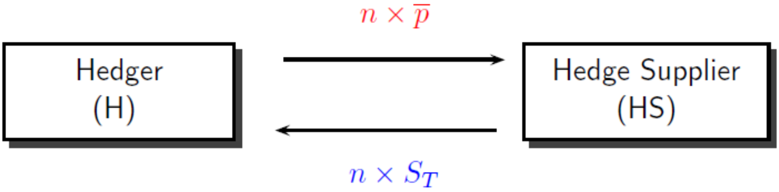
Questions

- collateral vs. capital
- pricing/valuation with bilateral default risk and collateral
- longevity swaps vs. IRSs

BESPOKE SOLUTIONS

Stylized example: single payment at time T

- notional n, fixed payment $\overline{p} \in (0,1)$
- variable payment S_T (realized survival rate)



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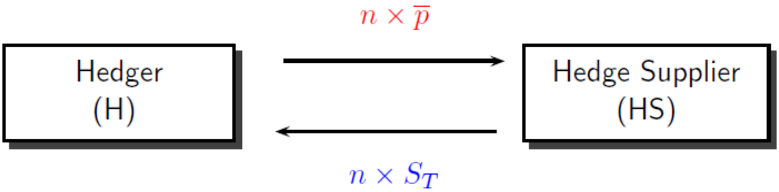

Swap value (hedger's viewpoint)

$$V_0 = nE^{\mathbb{Q}}\left[\exp\left(-\int_0^T r_t \mathrm{d}t\right)(S_T - \overline{p})\right]$$

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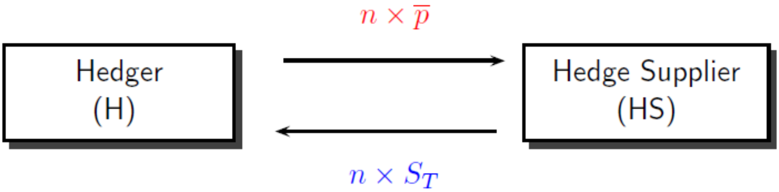

Longevity swap rate

$$\overline{p} = E^{\mathbb{Q}}\left[S_T\right] + \frac{\text{Cov}^{\mathbb{Q}}\left(\exp\left(-\int_0^T r_t \text{d}t\right), S_T\right)}{E^{\mathbb{Q}}\left[\exp\left(-\int_0^T r_t \text{d}t\right)\right]}$$

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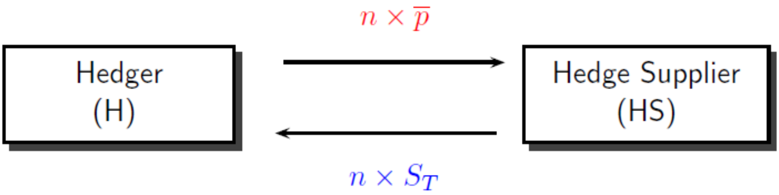

Longevity swap rate (r, S uncorrelated)

$$\overline{p} = E^{\mathbb{Q}}\left[S_T\right] + 0$$

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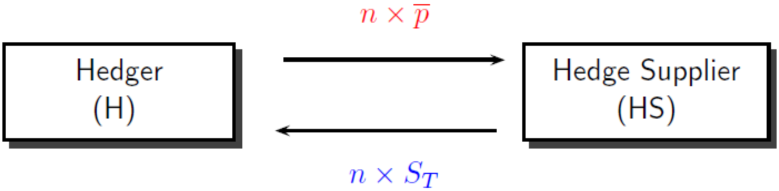


Longevity swap rate (r, S uncorrelated)

$$\overline{p} = E^{\mathbb{Q}}\left[S_T\right] + 0$$

Useful baseline case $\overline{p} = E^{\mathbb{P}}[S_T]$ (best estimate).

BACKTESTING

UK-based hedger

- 10,000 individuals (England & Wales) aged 65 in 1980
- indemnity-based solution over $1980 - 2005$
- interest rate risk hedged away

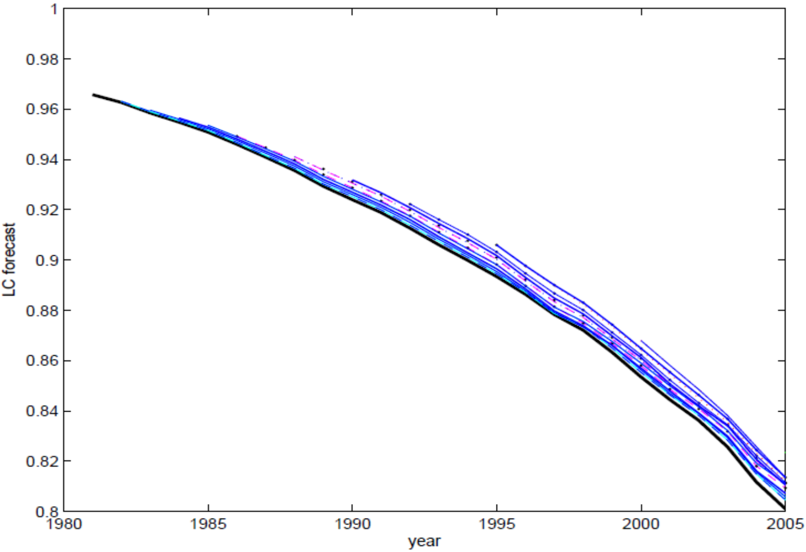
Realized cashflows

- population evolves as in Human Mortality Database (HMD)
- cashflow hedge in operation: $(realized\ rate) - (swap\ rate)$

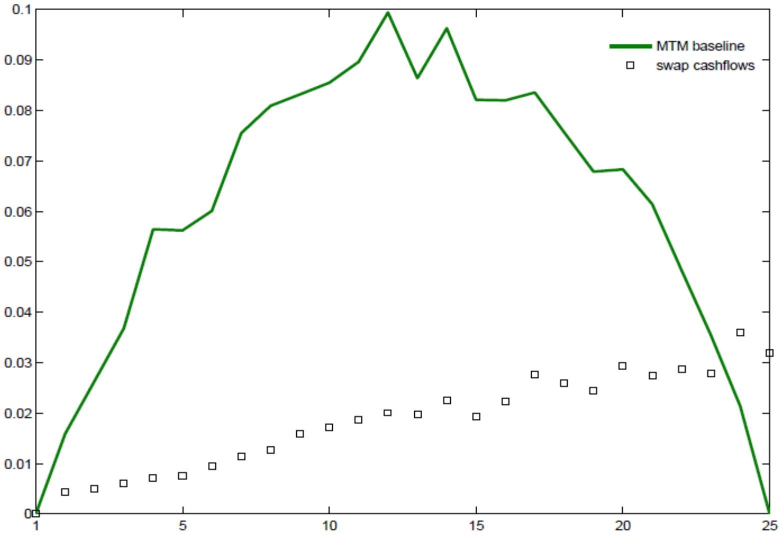
Marking to market/model (MTM)

- swap curves given by Lee-Carter forecasts based on most recent HMD data available

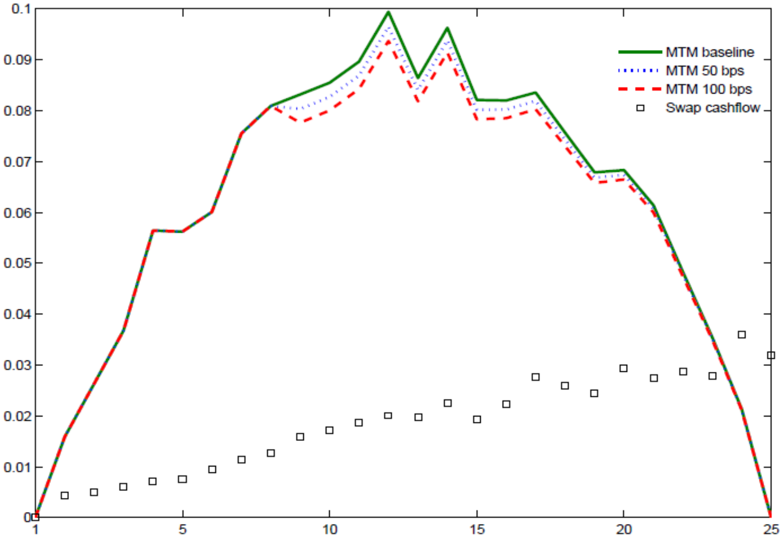
LONGEVITY SWAP RATES



CASHFLOWS AND MTM



HEDGE SUPPLIER'S CREDIT DETERIORATION



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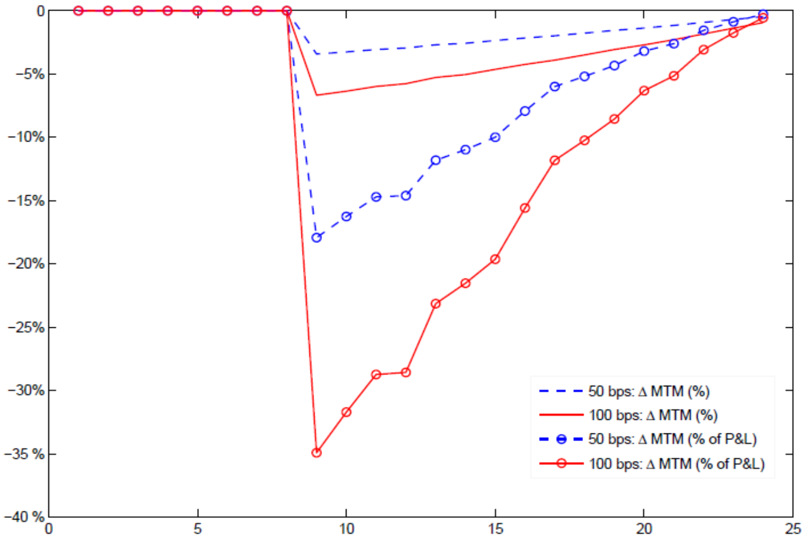

VALUATION

Hedger's viewpoint, cash collateral

- default intensities $(\lambda_t^h)_{t\geq 0}$, $(\lambda^{hs})_{t\geq 0}$
- collateral fraction, $(c_t)_{t\geq 0}$, of market value, $(V_t)_{t\geq 0}$
- $c_t V_t$ amount held (if pos.) or posted (if neg.)

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  - ⋆ **funding** cost
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- asymmetry in c, δ possible

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- asymmetry in c, δ possible
- swap market value (Biffis/al., 2011; Duffie/Huang, 1996; Brigo/al., 2008-)

$$V_0 = E^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + \Gamma_t)\mathrm{d}t\right)(S_T - \overline{p}^c)\right]$$

$$\Gamma_t := \begin{cases} (1 - c_t^h)\lambda_t^h - \delta_t^h c_t^h & \text{if } V_t < 0 \\ (1 - c_t^{hs})\lambda_t^{hs} - \delta_t^{hs} c_t^{hs} & \text{if } V_t \geq 0 \end{cases}$$

FULLY FLEDGED CALIBRATION

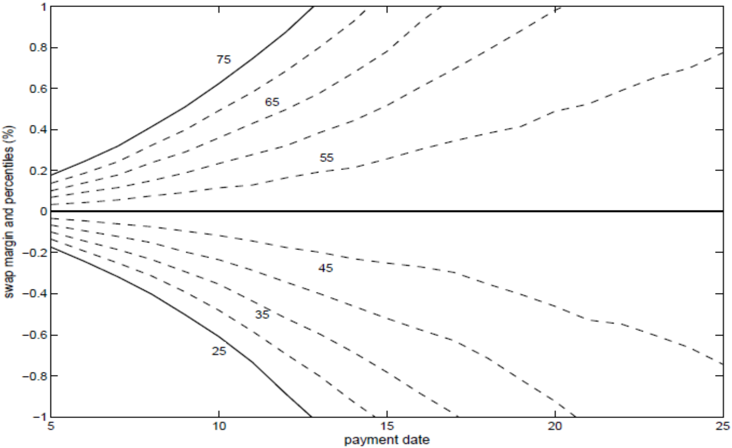
Building blocks

- two-factor short rate model
- TED spread for λ^{hs}
- $\lambda^h = \lambda^{hs} + \Delta$, $\Delta > 0$
- mortality: Lee-Carter mortality model
- implied IRS collateral costs (Johannes/Sunadersan 2007)

Two approaches to collateral net costs δ^h, δ^{hs}

1. funding costs associated with collateral flows
2. simulate Solvency II capital charges (1-year 99.5% VaR) accruing from representative longevity-linked liability, then use $Libor + 6\%$ or 12% for cost of capital charges

LONGEVITY SWAP MARGINS



Swap margins, $\frac{\overline{p}^c}{E^{\mathbb{P}}[S_T]} - 1$, against Lee-Carter mortality improvements quantiles.

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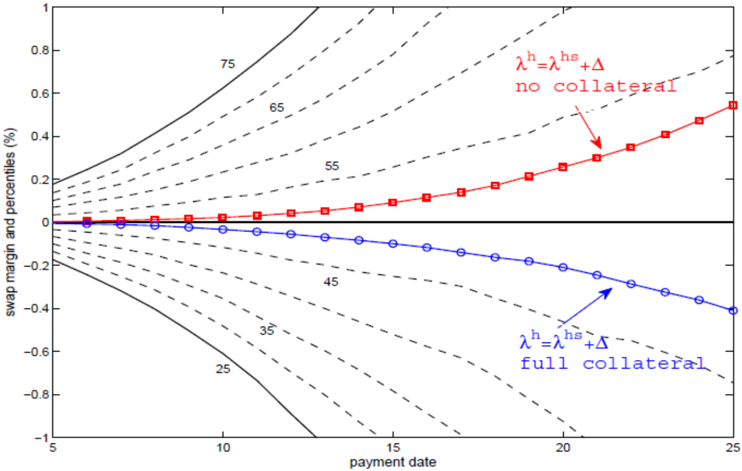

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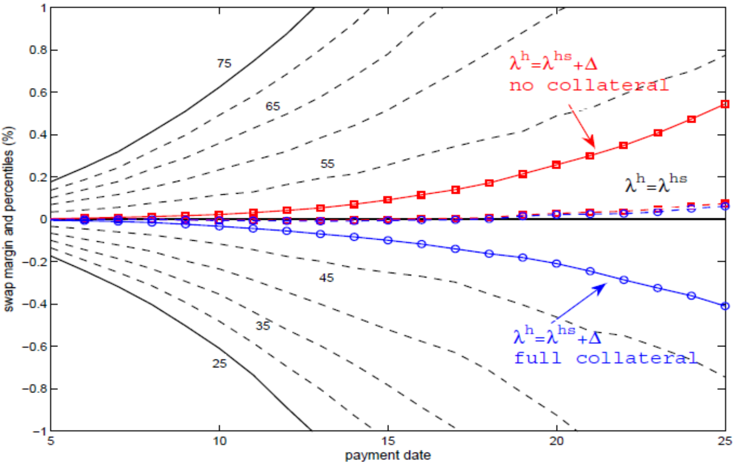

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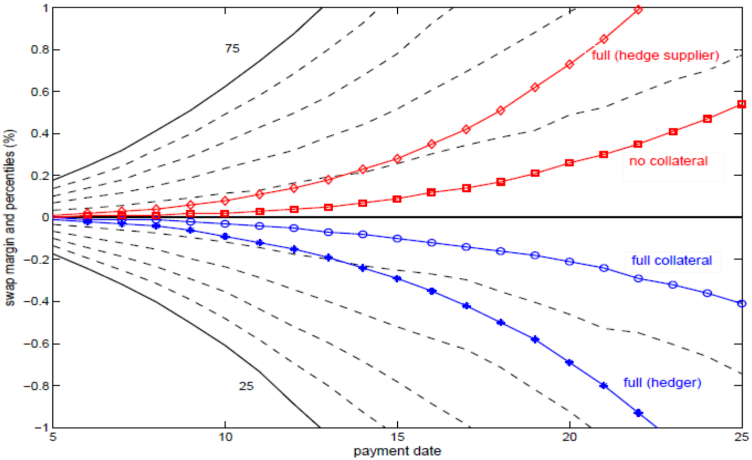

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LONGEVITY SWAP SPREADS

Swap spreads (basis points), $\overline{p}_T^c - E^{\mathbb{P}}[S_T]$

$\lambda^h = \lambda^{h,hs} + \Delta$ $\delta^{h,hs} = \delta$	Maturity payment (yrs)	$c^h = 0$ $c^{hs} = 0$ (bps)	$c^h = 0$ $c^{hs} = 1$ (bps)	$c^h = 1$ $c^{hs} = 0$ (bps)	$c^h = 1$ $c^{hs} = 1$ (bps)
	15	0.03	11.34	-11.76	0.05
$\Delta = 0$	20	1.11	19.93	-17.94	0.86
	25	1.50	21.25	-18.35	1.24
	15	5.45	16.79	-17.29	-5.84
$\Delta = 100$ bps	20	10.16	28.95	-27.08	-8.23
	25	10.96	30.75	-27.76	-9.19
	15	11.30	22.29	-22.90	-11.25
$\Delta = 200$ bps	20	19.26	38.06	-36.16	-17.42
	25	19.46	40.27	-37.02	-18.38

COMPARISON WITH THE IRS MARKET

	Maturity payment (yrs)	IRS $c^h = 0$ $c^{hs} = 1$ (bps)	IRS $c^h = 1$ $c^{hs} = 0$ (bps)	IRS $c^h = 1$ $c^{hs} = 1$ (bps)	longevity $c^h = 0$ $c^{hs} = 1$ (bps)	longevity $c^h = 1$ $c^{hs} = 0$ (bps)	longevity $c^h = 1$ $c^{hs} = 1$ (bps)
$\Delta = 0$	15	-7.96	-44.97	-52.86	11.34	-11.76	0.05
	20	-12.68	-42.64	-56.22	19.93	-17.94	0.86
	25	-17.94	-40.98	-58.92	21.25	-18.35	1.24
$\Delta = 100$ bps	15	-8.00	-67.87	-75.23	16.79	-17.29	-5.84
	20	-12.65	-63.84	-77.42	28.95	-27.08	-8.23
	25	-17.65	-60.63	-77.64	30.75	-27.76	-9.19

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CONCLUSION

Stochastic mortality models

- make sure (implicit) assumptions meet your needs
- consistency across applications is important and can be achieved

Mortality risk premia

- "standard" assumptions should not be taken at face value
- you can get a lot of mileage from restrictive models
- endogenizing risk premia through real world frictions (e.g., signalling, funding costs) is likeley to require much richer structure than usually assumed

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More details and references available in (see `www.ssrn.com`)

- Biffis/Denuit/Devolder (2010), Stochastic mortality under measure changes
- Biffis/Blake (2010a), Securitizing and tranching longevity exposures
- Biffis/Blake/Pitotti/Sun (2011), The cost of counterparty risk and collateralization in longevity swaps
- Biffis/Blake (2011), Informed intermediation of longevity exposures

THANK YOU FOR YOUR ATTENTION