

ROBUST HEDGING OF LONGEVITY RISK

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Hannover, 2012

Plan

- Intro + model
- Recalibration risk – introduction
- Robustness questions – index hedging
- Are some hedging instruments more robust than others?
- Static **Delta** and *Nuga* hedging
- Discussion

Background

- Annuity providers and pension plans
- Exposure to longevity risk
 - **systematic risk** (underlying mortality rates)
 - binomial risk (lives)
 - concentration risk (amounts)
- Alongside: interest rate risk, equity risk

Hedging problem 1

Annuity provider seeks to hedge its exposure to longevity risk

- Large cohort aged 65 at time 0
- Equal, level annuities payable for life
- $S(t, 65)$ = proportion still alive at t
- $PV = \sum_{t=1}^{\infty} e^{-rt} S(t, 65)$
- Objective: Hedge longevity risk in PV

Hedging problem 2

Annuity provider seeks to hedge its exposure to longevity risk

- Large cohort aged 65 at time 0
- Equal, level annuities payable for life
- $S(t, 65)$ = proportion still alive at t
- Deficit $D(t) = MCV_{Liabs}(t) - MCV_{Assets}(t)$
- Objective: Hedge longevity risk in $D(T)$
e.g. $T = 1$ under Solvency II

Hedging problem 3

Pension plan

- Cohort now aged 55
- Plan will buy annuities at age 65
- Objective: hedge the longevity risk in the annuity price

Options for hedging

- Customised hedges:
 - e.g. longevity swap
 - floating leg linked to OWN cashflows
 - indemnification
- Index-based hedges:
 - Standardised contracts
 - e.g. Linked to a national index

Focus of this talk

Index-based hedges

- Customised longevity swaps only available to very large pension plans
- Index-based hedges
 - smaller schemes
 - better value for money for large plans ???
 - Quantity of hedging instrument
 - Hedge effectiveness
 - Price

How confident are we in these quantities? \Rightarrow ROBUSTNESS

Simple example

- Static *value* hedge: $t = 0 \longrightarrow T$
- $a_k(T, x)$ = population k annuity value at T
- Liability value $L(T) = a_2(T, 65)$
- Hedging instrument: deferred longevity swap

$$H(T) = a_k(T, x) - \hat{a}_k^{\text{fxd}}(0, T, x)$$

$\hat{a}_k^{\text{fxd}}(0, T, x)$ = value at T of swap fixed leg

- $k = 2$ (CMI) \Rightarrow CUSTOMISED hedge
- $k = 1$ (E&W) \Rightarrow INDEX hedge

Hedging: basic idea

- L = liability value
- H = value of hedging instrument
- Objective: minimise $Var(\text{deficit}) = Var(L + hH)$

$$\Rightarrow \text{hedge ratio, } h = -\frac{Cov(L, H)}{Var(H)} = -\rho \frac{S.D.(L)}{S.D.(H)}$$

$$\text{Hedge effectiveness} = 1 - \frac{Var(L + hH)}{Var(L)} = \rho^2$$

More general: multiple assets

$$\Rightarrow \text{minimise } Var(L + h_1H_1 + \dots + h_nH_n)$$

Simple example: APC model (Cairns et al., 2011a)

$m_k(t, x)$ = population k death rate

$$\log m_k(t, x) = \beta^{(k)}(x) + \kappa^{(k)}(t) + \gamma^{(k)}(t - x)$$

$\beta^{(1)}(x), \beta^{(2)}(x)$ population 1 and 2 age effects

$\kappa^{(1)}(t), \kappa^{(2)}(t)$ period effects; mean reverting spread

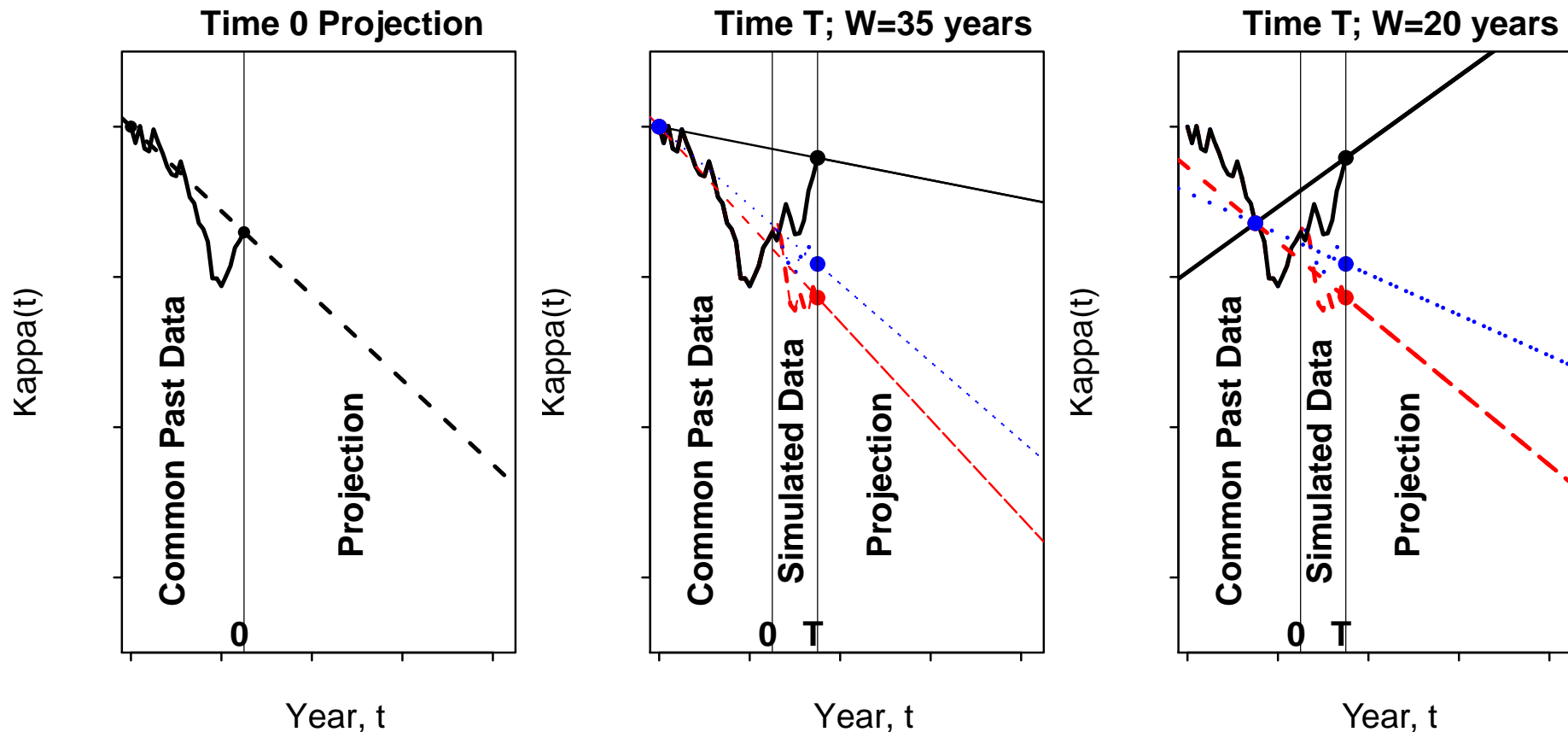
$\gamma^{(1)}(c), \gamma^{(2)}(c)$ cohort effects

Key: $\nu_\kappa = \kappa^{(1)}(t), \kappa^{(2)}(t)$ long term trend

Realism: valuation model \neq simulation model

- (Re-)calibration using data up to $T \Rightarrow$ **realistic!**
- Valuers just observe historical mortality plus
one future sample path of mortality from 0 to T
 \Rightarrow do not know the “true” simulation/true model
- Using true model \Rightarrow too optimistic (??) c.f. Black-Scholes
- Valuation model + calibration window \Rightarrow *Knightian*
Uncertainty

Recalibration risk – example (random walk)



- You *will* recalibrate at T
- Recalibration depends on as yet unknown experience from 0 to T
- Recalibration depends on length of lookback window

Hedge Effectiveness: (Cairns et al., 2011b; Longevity 6)

Key conclusions: index-based hedging

- Recalibration \Rightarrow risk \nearrow
- BUT hedge effectiveness also \nearrow

WHY?

Additional trend risk is common to both populations.

$$a_k(T, x) \approx f(\beta_{[x]}^{(k)}, \kappa_T^{(k)}, \gamma_{T-x+1}^{(k)}, \nu_{\kappa})$$

Robustness

How robust are estimates of:

- Optimal hedge ratios h_1, \dots, h_n
- Hedge effectiveness
- Initial hedge instrument prices $\pi(H_1), \dots, \pi(H_n)$

... relative to ...

Robustness

How robust are key quantities relative to

- Treatment of parameter risk
- Treatment of population basis risk
- Valuation model: recalibration risk
- Poisson risk
- Use of latest EW data
- Simulation model + calibration

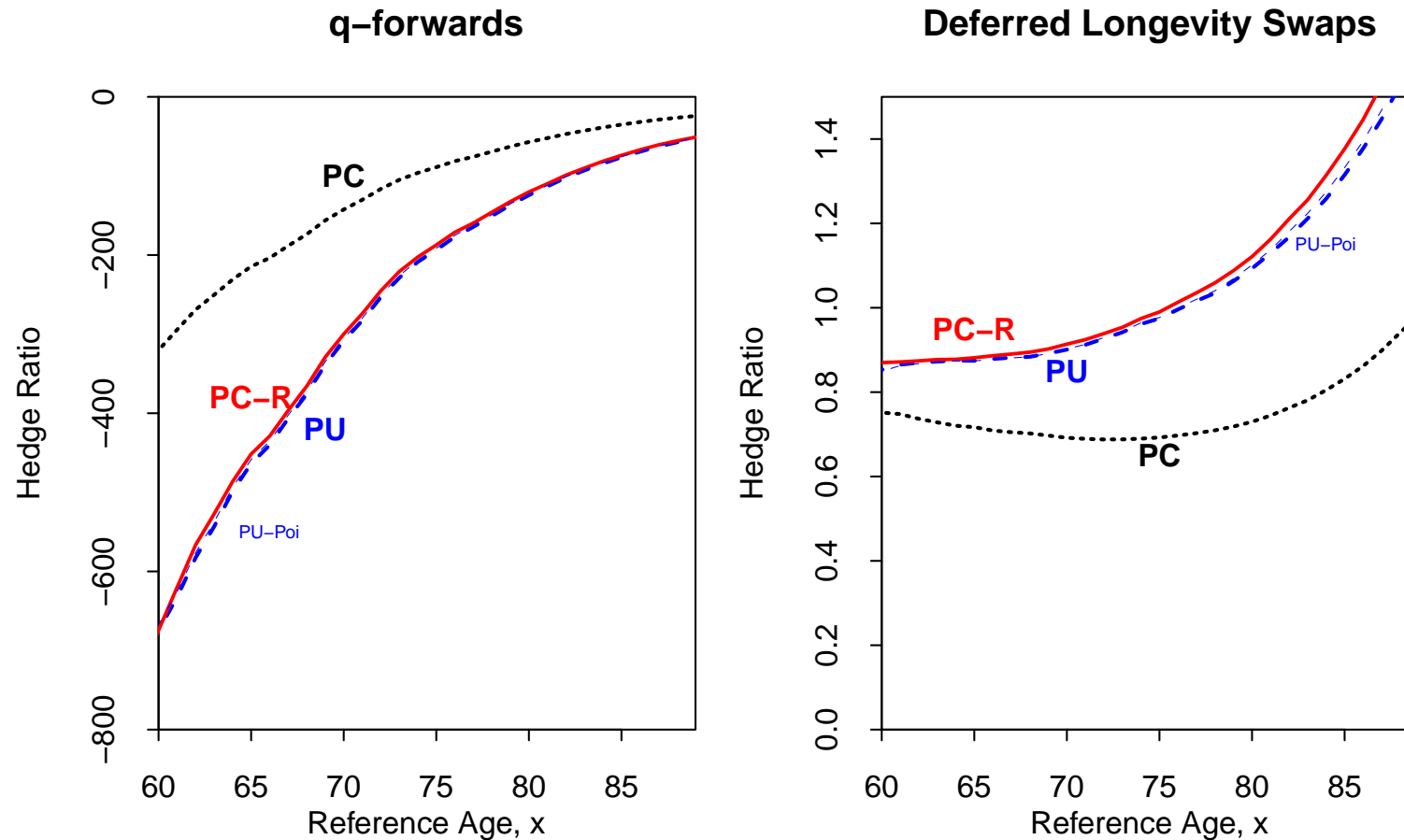
Modelling Variants

- PC: Full parameter certainty (PC);
Valuation Model NOT recalibrated in 2015
- PC-R: As full PC
Except: Valuation Model recalibrated in 2015
- PU: Full parameter uncertainty with recalibration
- PU-Poi: Full PU with recalibration + Poisson risk

Hedging options

- Recall: Liability, $L = a_2(T, 65)$ (CMI)
- Hedging instrument (ref England & Wales):
 - $H = a_1(T, x) - a_1^{\text{fxd}}(0, T, x)$
 - OR
 - q -Forward maturing at T (www.LLMA.com)
 - $H = q_1(T, x) - q_1^F(0, T, x)$
- Both cases: for a range of reference ages x

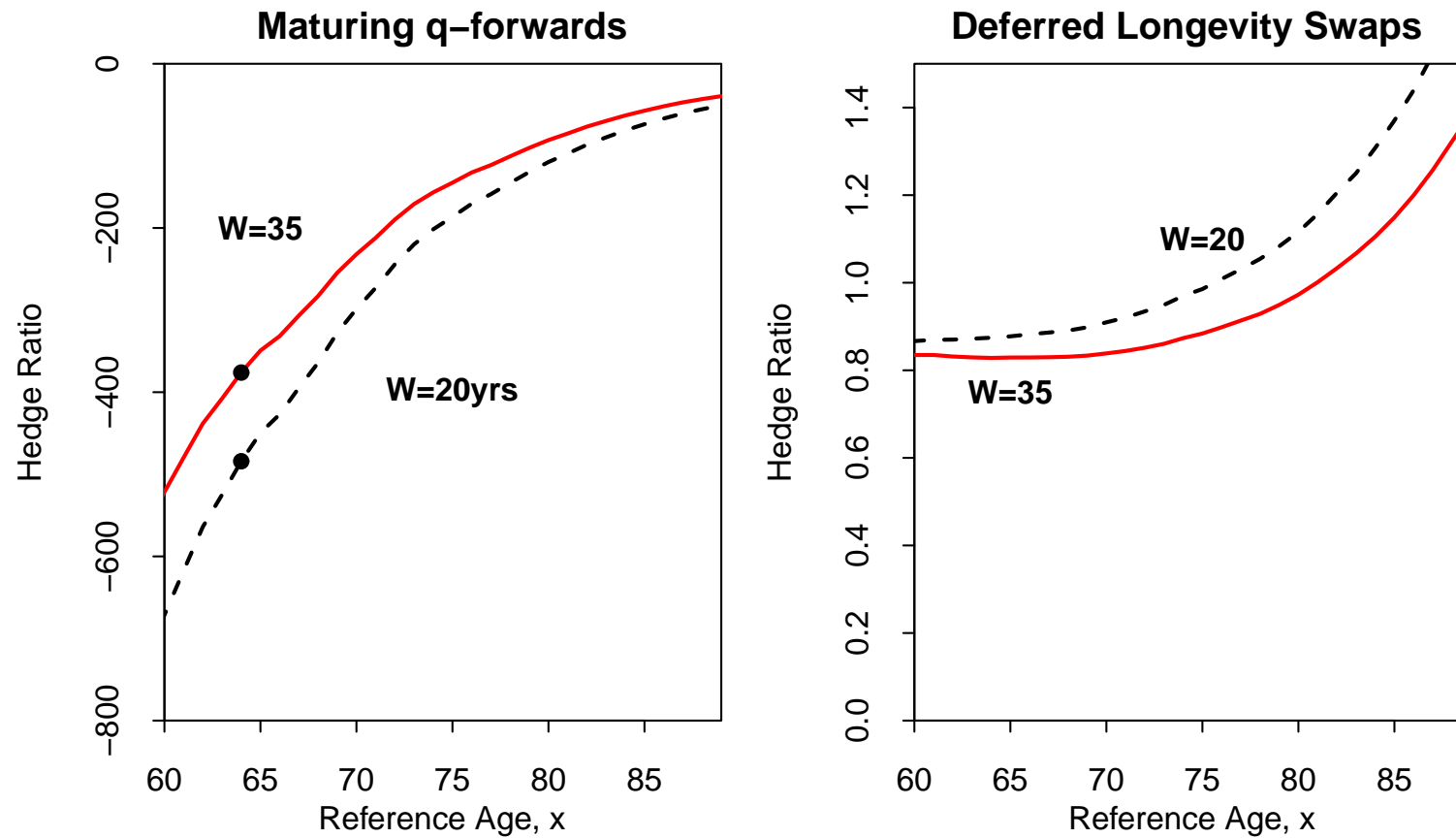
Robustness of Hedge Ratios



PC \rightarrow PC-R not robust; PC-R \rightarrow PU robust

deferred longevity swaps better than maturing q -Forwards

Robustness relative to recalibration window, W



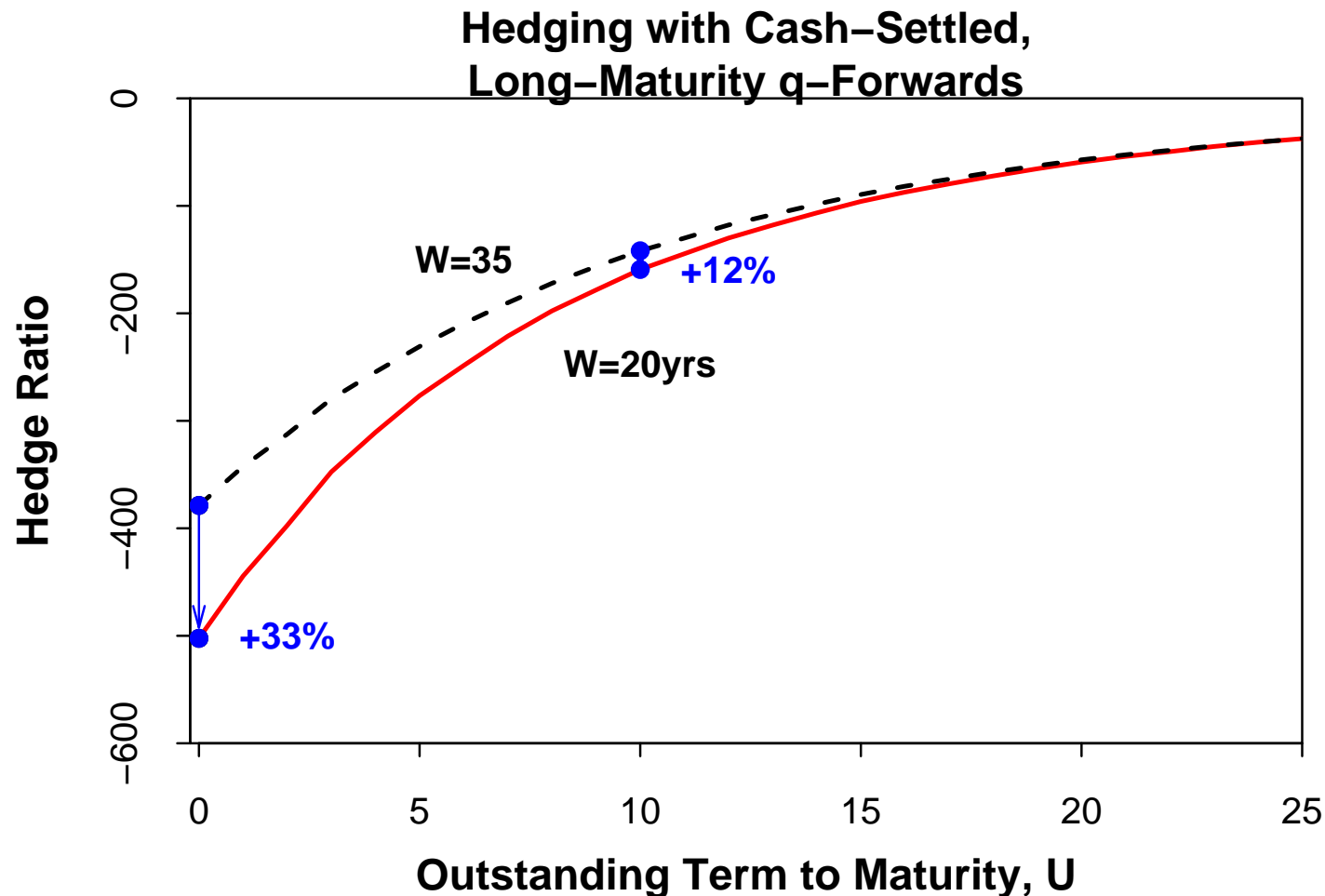
Deferred longevity swaps better than maturing q -Forwards

Robustness relative to recalibration window, W

Longevity swaps are more robust:

- Liability, L , and longevity swap, H , depend on
 - $\kappa_T^{(1)}$ and ν_κ
 - BUT in differing proportions \Rightarrow single H not robust
- Maturing q -Forward depends on $\kappa_T^{(1)}$ only
 - \Rightarrow even less robust
- Possible market solution:
 - $(0, T + U, x)$ q -Forward, cash settled at T

Robustness relative to recalibration window, W



$T + U$ q -Forward is cash settled at time T

\Rightarrow value depends on $\kappa_t^{(1)}$ and ν_κ

Robustness relative to recalibration window, W

- If we know W , then ν_κ linear in $\kappa_T^{(1)}$
 \Rightarrow one hedging instrument sufficient
- If W is not known
or, ν_κ determined by other methods
 \Rightarrow two hedging instruments are required
 \Rightarrow Delta and “Nuga” hedging

Delta and Nuga Hedging

Recall: $a_k(T, x) \approx f(\beta_{[x]}^{(k)}, \kappa_T^{(k)}, \gamma_{T-x+1}^{(k)}, \nu_\kappa)$

Liability: $L = a_2(T, x)$.

Hedge instruments:

$$H_1 = q_1(T, x_1) - q_1^{\text{fxd}}(0, T, x_1) \rightarrow h_1 \text{ units}$$

$$H_2 = q_1(T + U, x_2) - q_1^{\text{fxd}}(0, T + U, x_2) \rightarrow h_2 \text{ units}$$

(H_2 cash settled at T)

Delta and Nuga hedging \Rightarrow require

$$\text{Deltas: } \alpha \frac{\partial L}{\partial \kappa^{(2)}} = -h_1 \frac{\partial H_1}{\partial \kappa^{(1)}} - h_2 \frac{\partial H_2}{\partial \kappa^{(1)}}$$

$$\text{and Nugas: } \frac{\partial L}{\partial \nu_\kappa} = -h_1 \frac{\partial H_1}{\partial \nu_\kappa} - h_2 \frac{\partial H_2}{\partial \nu_\kappa}$$

where $\alpha = \text{Cov}(\kappa_T^{(1)}, \kappa_T^{(2)}) / \text{Var}(\kappa_T^{(1)})$.

Concept:

same idea as Vega hedging in equity derivatives

– hedging against changes in a parameter that is supposed to be constant.

Numerical example: $L = a_2(T, 65)$, $T = 10$

Four strategies:

A: No hedging

B: H_1 only; h_1 optimal for $W = 20$

C: H_1 only; h_1 optimal for $W = 35$

D: H_1 and H_2 ; Delta and Nuga hedging

Numerical example: $L = a_2(T, 65)$, $T = 10$

Strategy	q-F($T, 64$)	q-F($T + T, 74$)	$Var(\text{Deficit})$	Hedge Eff.	
	h_1	h_2			
$W = 20$					
A	0	0	0.3481	0	
B	500.7	0	0.03435	0.9013	(1)
C	389.0	0	0.04996	0.8565	(3)
D	-279.6	256.4	0.03797	0.8909	(2)
$W = 35$					
A	0	0	0.2233	0	
B	500.7	0	0.04953	0.7782	(3)
C	389.0	0	0.03392	0.8481	(1)
D	-279.6	256.4	0.03493	0.8436	(2)

Numerical example: discussion

- Nonlinearities $\Rightarrow D < B$ instead of $D = B$
- BUT
 - $W = 20 \Rightarrow$
 - D is nearly optimal
 - C is much worse
 - $W = 35 \Rightarrow$
 - D is nearly optimal
 - B is much worse

Robustness relative to other factors

Results are robust relative to:

- inclusion of parameter uncertainty in $\beta_x^{(k)}$, $\kappa_t^{(k)}$, $\gamma_c^{(k)}$
- pension plan's own small-population Poisson risk
- index population: EW-size Poisson risk, maybe smaller
- CMI data up to 2005 + EW data up to 2005

versus

CMI data up to 2005 + EW data up to 2008

Conclusions

Robust hedging requires *inclusion* of

- Recalibration risk (Nuga)
- Careful treatment of recalibration window
- Long-dated hedging instruments to handle *Nuga* risk

Results appear to be robust relative to

- Poisson risk
- Parameter uncertainty (other than recalibration risk)
- Treatment of latest data

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