Price contagion through balance sheet linkages

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Talk Outline

1. Introduction
2. The Model
3. Asset prices
4. Systemicness of leverage targeting
5. The network of asset prices
6. Policy implications and concluding remarks
Systemic Risk

- **Systemically** important institutions can contaminate others and spiral into shocks destabilizing the financial system.
- Current literature has put forward two main approaches for modeling systemic risk:
  - **Bottom-up approach:**
    - Model linkages arising when financial entities are connected via direct bilateral exposures
    - Often used towards descriptive characteristics of system via simulation with little analytical results
  - **Top-down approach:**
    - Global indicators of systemic risk and contribution of each financial institution
    - Granularity of system not as fine compared to bottom-up, but generally leads to more analytically tractable predictions
Top-down approach

- Spread of distress across entities captured by systemic risk measures.
- Noticeable contributions include:
  - Adrian and Brunnermeier (2001): CoVar relates systemic risk contribution of an entity to value at risk of the overall system
  - Acharaya et al. (2012): systemic expected shortfall index to measure expected amount of undercapitalization under systemic events
  - Brownless and Engle (2014): SRISK to measure expected capital shortfall under prolonged period of market distress
Balance sheet contagion vs Network Models

- Network studies assume asset prices fixed at their book values: balance sheets only take hits at default events.
- Empirical evidence suggests that financial institutions react to asset price changes by actively managing their balance sheets.
- **Distress propagation**: forced sales of illiquid assets may depress prices, and prompt financial distress at other banks with similar holdings.
- Adrian and Shin (2008): *if the domino model of financial contagion were the relevant one, defaults on products such as subprime mortgages would have had a much smaller impact.*
Our contribution

- Top-down model aiming at quantifying price linkages arising when firms, holding similar assets on their balance sheets, manage their leverage ratios to conform with pre-specified target levels.

- Related contributions include:
  - Shleifer and Vishny (2011): how asset fire sales lead to downward spirals or cascades in asset prices
  - Brunnermier and Pedersen (2009): interaction of market and funding liquidity
  - Greenwood, Landier and Thesmar (2014): how distribution of bank leverage and risk exposures lead to formation of systemic risk
  - Cont et al (2015): fire sales and amplification effects arising when banks need to maintain minimal leverage requirements
Motivating Evidence

- Empirical studies indicate that banks actively manage leverage
  - Gropp and Heider (2010) find that banks adjust toward their target leverages at fast speeds
  - Berger et al. (2008) find that poorly capitalized banks adjust toward their targets more quickly than well capitalized banks
  - Adrian and Shin (2008, 2010) find that commercial banks track their leverage ratios, while investment banks even have procyclical leverage
  - Adrian and Shin (2010): micro-foundation of leverage procyclicality driven by value at risk
Adrian and Shin (2010)

Figure 2.3: Total Assets and Leverage of Commercial Banks

Figure 2.4: Total Assets and Leverage of Security Brokers and Dealers
Market Participants and Assets

- Market divided into two sectors:
  - Banking: commercial and investment banks.
  - Non-banking: mutual, money market and pension funds, insurances.
- Banks manage their leverage ratios to conform with a pre-specified target level.
- Non-banking sector is not subject to stringent leverage management.
Balance sheet management

- State of each bank described by its balance sheet which consists of assets, equity and debt.
- Banks manage their capital structure by buying and selling assets, and by increasing or reducing their level of debt.
- Banks do not raise new equity capital in response to a positive or negative shock to the asset value.
Figure 3. Scatter chart of \((\Delta A_t, \Delta E_t)\) and \((\Delta A_t, \Delta D_t)\) for changes in assets, equity and debt of US investment bank sector consisting of Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch and Morgan Stanley between Q1:1994 and Q2:2011 (Source: SEC 10Q filings).

Figure 4. Scatter chart of \((\Delta A_t, \Delta E_t)\) and \((\Delta A_t, \Delta D_t)\) for changes in assets, equity and debt of US commercial bank sector at \(t\) between Q1:1984 and Q2:2010 (Source: FDIC call reports).
The Banking sector

Assets of banking sector

- $K$ types of assets. Market prices at $t$ denoted by $P^k_t$:

$$P_t = (P^1_t \ P^2_t \ \cdots \ P^K_t)^\top$$

- The aggregate supply of each asset is fixed and given by

$$Q_{\text{tot}} = (Q^1_{\text{tot}} \ \cdots \ Q^K_{\text{tot}})^\top$$

- $N$ banks. Holdings of bank $i$ are given by

$$Q^i_t = (Q^{1i}_t \ Q^{2i}_t \ \cdots \ Q^{Ki}_t)^\top.$$  

- Market value of the $i$:th bank’s holdings of asset $k$ is $A^k_{ti} = P^k_t Q^{ki}_t$. Then

$$A^i_t = (A^{1i}_t \ A^{2i}_t \ \cdots \ A^{Ki}_t)^\top.$$
Debt of bank $i$ at time $t$ is $D_t^i$.

Key behavioral assumption: each bank $i$ tracks a fixed target leverage $\lambda_i$.

This yields the leverage equation:

$$\frac{D_t^i}{1^\top A_t^i - D_t^i} = \lambda_i$$

Interest rate $r = 0$ to simplify exposition.
The Banking sector

Bank’s operations

- Each bank receives revenues $\Delta R^i_t$, net of operating costs and dividends, over each time interval $[t, t + \Delta t]$.

- Cash flows result from:
  1. operating revenues
  2. increase or reduction in debt used by the bank to either purchase more assets or liquidate part of its current holdings.

- Cash flow allocation strategy

  \[ \alpha_t^i = (\alpha_t^{1i} \cdots \alpha_t^{Ki})^\top, \quad \alpha_t^{ki} > 0 \]

  with \[ \sum_{k=1}^{K} \alpha_t^{ki} = 1 \]
The fundamental cash flow equation is

\[ P_{t+\Delta t}^k \Delta Q_{t}^{ki} = \alpha_t^{ki} \left( \Delta R_t^i + \Delta D_t^i \right) \]

- LHS: amount invested by bank \( i \) in asset \( k \) at \( t + \Delta t \)
- RHS: proceeds from operating revenues and debt issuance in time interval \([t, t + \Delta t]\)
Combining leverage ratio and cash flow equation leads to

$$\Delta Q_{t}^{ki} = \frac{\alpha_{t}^{ki}}{P_{k}^{t+\Delta t}} \left( \lambda_i Q_i^{t} \Delta P_t + (1 + \lambda_i) \Delta R_i^t \right)$$

In the absence of revenue shocks and if $\Delta P_{t}^{h} = 0$ for $h \neq k$

$$\frac{\Delta Q_{t}^{ki}}{Q_{t}^{ki}} = \lambda_i \alpha_{t}^{ki} \frac{\Delta P_{t}^{k}}{P_{k}^{t+\Delta t}}$$
The Banking sector

Upward sloping demand

\[ \frac{\Delta Q_t^{ki}}{Q_t^{ki}} = \lambda_i \alpha_t^{ki} \frac{\Delta P_t^k}{P_t^k + \Delta t} \]

- \( \lambda_i \alpha_t^{ki} \) can be interpreted as price elasticity.
- Suppose the price of one unit of the \( k \)-th asset rises from \( t \) to \( t + \Delta t \).
- The firm increases its debt level to track leverage ratio,
- The firm invests the raised capital by purchasing asset units.
Non-banking demand

- Non-bank institutions also trade in the available assets
- Non-banking sector holds a quantity $Q_{t}^{k,\text{nb}}$ of asset $k$ at $t$, $A_{t}^{k,\text{nb}} = P_{t}^{k} Q_{t}^{k,\text{nb}}$
- The incremental demand is given by

$$\Delta Q_{t}^{k,\text{nb}} = \frac{\gamma_{k}}{P_{t+\Delta t}^{k}} Q_{t}^{k,\text{nb}} \left( \Delta Z_{t}^{k} - \Delta P_{t}^{k} \right)$$

where $\Delta Z_{t}^{k}$ are asset-specific demand shocks
The Non-banking sector

Downward-sloping demand

- Assume no leverage ratio trackers. Market clearing leads to asset price dynamics $\Delta P^k_t = \Delta Z^k_t$.

- Relationship between price and demand conditional on $\Delta Z^k_t = 0$ given by

$$\frac{\Delta Q^{k,\text{nb}}_t}{Q^{k,\text{nb}}_t} = -\gamma_k \frac{\Delta P^k_t}{P^k_{t+\Delta t}}$$

- $\gamma_k > 0$: elasticity of nonbanking demand for asset $k$
Notations and Definitions

- Given vectors \( u = (u_1 \cdots u_n)^\top \) and \( v = (v_1 \cdots v_n)^\top \)
  - The componentwise product is \( u \circ v = (u_1 v_1 \cdots u_n v_n)^\top \)
  - The componentwise ratio is \( \frac{u}{v} = \left( \frac{u_1}{v_1} \cdots \frac{u_n}{v_n} \right)^\top \)

- \( \text{Diag}(u) \) is the diagonal matrix with \( u \) on the diagonal.
Systemicness matrix

- The systemicness matrix $S_t$ is given by

$$S_t = \sum_{i=1}^{N} \frac{\alpha^i_t}{\gamma \circ A^n_{t}} \lambda_i A^i_{t}^\top$$

- Its componentwise form is

$$S^{k\ell}_t = \sum_{i=1}^{N} \frac{\alpha^{ki}_t}{\gamma_k A^{k,i}_{t,nb}} \lambda_i A^\ell_{i,t}$$

- Key determinant for excess price correlation induced by the leverage targeting banks.
Graph based interpretation

- Systemicness matrix $S_{t}^{k\ell} = \sum_{i=1}^{N} \alpha_{t}^{ki} \frac{\lambda_{i}A_{t}^{\ell i}}{\gamma_{k}A_{t}^{k,\text{nb}}} \text{ interpreted as weighted adjacency matrix of network:}$
  - Directed edge from node $\ell$ to node $k$ with weight $S_{t}^{k\ell}$.
  - Return shock to asset $\ell$ of size $y_{\ell}$ propagates along the edge $(\ell, k)$ and results in a shock to asset $k$ of size $y_{k} = S_{t}^{k\ell} y_{\ell}$
  - The shock $y_{k}$ forces further leverage adjustments, causing return shocks to other assets, and so on.
Market dynamics

Proposition

The cash flow equation, leverage equation, the non-banking demand function, and market clearing imply

\[
\begin{align*}
\frac{\Delta P_t}{P_t} &= (I - S_t)^{-1} \left[ \frac{\Delta Z_t}{P_t} + \sum_{i=1}^N \frac{\alpha^i_t}{\gamma \circ A^\text{nb}_t} (1 + \lambda_i) \Delta R^i_t \right] \\
\Delta A^i_t &= Q^i_t \circ \Delta P_t + \alpha^i_t \left( \lambda_i Q^i_t \top \Delta P_t + (1 + \lambda_i) \Delta R^i_t \right) \\
\Delta A^\text{nb}_t &= Q^\text{nb}_t \circ \left( \gamma \circ \Delta Z_t - (\gamma - 1) \circ \Delta P_t \right)
\end{align*}
\]

assuming that the matrix inverse exists.
Suppose all eigenvalues of $S_t$ are less than one. Then

$$\frac{\Delta P_t}{P_t} = [I + S_t + S_t^2 + S_t^3 + \ldots] \Delta Y_t$$

- **Direct impact**: first term $I$ in the power series expansion
- **Indirect** effect: Suppose negative shock occurs.
  - Term $S_t$ corresponds to first round of deleveraging.
  - Term $S_t^2$ corresponds to a second round of deleveraging.
  - Each round impacts prices and total realized return is the aggregate outcome of this process
The spectral radius of the systemicness matrix

The spectral radius \( \rho(S_t) \), \( S_t = \sum_{i=1}^{N} \frac{\alpha_i^t}{\gamma^o A_t^{nb}} \lambda_i A_i^{i^T} \), is the aggregate level of vulnerability in the system.

**Proposition**

Assume all asset holdings and all cash flow allocation weights \( \alpha_{ki}^t \) are nonnegative. We have the bounds

\[
\max_{k=1,\ldots,K} \frac{\sum_{i=1}^{N} \lambda_i \alpha_{ki}^t A_{ki}^t}{\gamma_k A_t^{k, nb}} \leq \rho(S_t) \leq \max_{k=1,\ldots,K} \frac{\sum_{i=1}^{N} \lambda_i \alpha_{ki}^t \mathbf{1}^T A_i^t}{\gamma_k A_t^{k, nb}}
\]
The upper bound is given by

\[
\max_{k=1,\ldots,K} \frac{\sum_{i=1}^{N} \lambda_i \alpha_t^k 1^\top A_t^i}{\gamma_k A_{t}^{k,\text{nb}}}
\]

- Quantifies the size of leverage targeting banks relative to the size of the nonbanking sector, as measured by elasticity-weighted assets.
- When nonbanking sector is large, \( \rho(S_t) \) is small, and the impact on realized returns is moderate.
The lower bound is given by

$$\max_{k=1,\ldots,K} \frac{\sum_{i=1}^{N} \lambda_{i} \alpha_{t}^{ki} A_{t}^{ki}}{\gamma_{k} A_{t}^{k, \text{nb}}}$$

- Numerator only involves the banks’ holdings of the $k$:th asset
- If the nonbanking sector is small in relative terms, $\rho(S_{t})$ is large, causing strong impact on realized returns
Fixed relative exposure strategy

- Relative exposure strategy is given by $\alpha_{ki}^t = A_{ki}^t / (1^\top A_t)$.
- Upper bound on the spectral radius given by

$$\rho(S_t) \leq \max_{k=1,...,K} \frac{\sum_{i=1}^N \lambda_i A_{ki}^t}{\gamma_k A_{k, nb}^t}$$

- Existence of an asset class for which holdings of highly levered banks are large relatively to the nonbanking sector destabilizes system.
- Relation to illiquidity concentration measure by Duarte and Eisenbach (2013).
Cash Flow Allocation Strategies

Liquidity based strategy I

- Banks sell (buy) the most liquid assets when there is a need to delever (lever up)

\[ \alpha_{ki}^t = \frac{\gamma_k A_{ki}^{k,\text{nb}}}{\sum_{\ell=1}^K \gamma_{\ell} A_{\ell}^{\ell,\text{nb}}} \]

- Spectral radius equals its upper bound, i.e.

\[ \rho(S_t) = \frac{\sum_{i=1}^N \lambda_i \mathbf{1}^\top A_i^i}{\sum_{k=1}^K \gamma_k A_t^{k,\text{nb}}} \]
Cash Flow Allocation Strategies

Liquidity based strategy II

- Spectral radius only depends on the relative *aggregate* size of the banking and the nonbanking sectors.
- Existence of an asset class for which price elasticity is high and for which the nonbanking sector is large relative to the banking sector, would be enough to stabilize the system.
- Severe deleveraging needs can be absorbed by the nonbanking sector. The remaining, potentially illiquid, assets are not subjected to fire sales.
Liquidity based strategy III

Proposition

Assume all asset holdings are nonnegative. Then

\[
\frac{\sum_{i=1}^{N} \lambda_i \mathbf{1}^\top A^i_t}{\sum_{k=1}^{K} \gamma_k A_{t, \text{nb}}^k} \leq \max_{k=1, \ldots, K} \frac{\sum_{i=1}^{N} \lambda_i A_{t}^{ki}}{\gamma_k A_{t, \text{nb}}^k}
\]

liquidity based

fixed exposure
One-step impact of shocks

- Deleveraging activities of bank $i$ contributes to one-step propagation of shocks from asset $\ell$ to asset $k$:
  \[
  \alpha_{ki}^i \frac{\lambda_i A_{i t}^{\ell}}{\gamma_k A_{t}^{k, nb}}
  \]

- Total one-step return impact caused by bank $i$:
  \[
  \lambda_i \mathbf{1}^\top A_{i t}^i \sum_{k=1}^{K} \alpha_{ki}^i \frac{1}{\gamma_k A_{t}^{k, nb}}
  \]

- Strongest one-step systemic impact by large banks that are highly levered and allocate their cash flows to illiquid assets
Aggregate impact of shocks

- One-step return impacts give an incomplete view of the systemic properties of the banking sector.
- Aggregate impact on return of asset $k$ due to a shock in asset $\ell$ given by

$$ (I - S_t)^{-1} = I + S_t + S_t^2 + \cdots $$

- Can higher-order effects beyond the one-step impact be neglected?
Illustrative Example I

- Three assets \((K = 3)\) and two banks \((N = 2)\).
- Both banks track the same debt-over-equity leverage ratio \(\lambda = 9\).
- Current assets and fixed relative exposure strategies given by

<table>
<thead>
<tr>
<th></th>
<th>Banking Sector</th>
<th>Non-Banking Sector</th>
</tr>
</thead>
</table>
| \(A^1_t\)        | \[
\begin{bmatrix}
1 \\
9 \\
0
\end{bmatrix}
\] | \[
\begin{bmatrix}
0.1 \\
0.9 \\
0
\end{bmatrix}
\] | \[
\begin{bmatrix}
900 \\
400 \\
300
\end{bmatrix}
\] |
| \(A^2_t\)        | \[
\begin{bmatrix}
0 \\
1 \\
4
\end{bmatrix}
\] | \[
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\] | \[
\begin{bmatrix}
900 \\
400 \\
300
\end{bmatrix}
\] |
| \(\alpha^1_t\)   | \[
\begin{bmatrix}
0.1 \\
0.9 \\
0
\end{bmatrix}
\] | \[
\begin{bmatrix}
0.2 \\
0.8
\end{bmatrix}
\] |
Illustrative Example II

- Systemicness matrix becomes

\[
S_t = \begin{bmatrix}
0.001 & 0.009 & 0 \\
0.02 & 0.187 & 0.018 \\
0 & 0.24 & 0.96
\end{bmatrix}, \quad \rho(S_t) = 0.97
\]

- \(S_t^{3,1} = 0\), but the (3, 1) entry of \((I - S_t)^{-1}\) is 0.17.
- A shock to asset 1 has a sizable impact on the return of asset 3, even though the one-step effect is zero!
Asset growth and extreme sensitive regime

Numerical setup

- $\Delta R_t^i \sim \mathcal{N}(0, 0.1)$, $\Delta Z_t^k \sim \mathcal{N}(0, 0.1)$
- Initial aggregate size of banking sector is 8% of the nonbanking sector
- $N = K = 2$ and $P_{01}^1 = P_{02}^2 = 1$
- $\lambda_1 = \lambda_2 = 10$ and $\gamma_1 = \gamma_2 = 9$
Fixed relative exposure strategy
Unstable realization

- Liquidity based strategy
- Fixed relative exposure strategy
Conclusions

Policies suggestions?

- Spectral radius of systemicness matrix characterizes stability of price dynamics in presence of leverage targeting banks.
- Stability improves if (i) the banking sector is small, (ii) the nonbanking sector is large, (iii) nonbanking demand is highly elastic, (iv) target leverage is low, or (v) banks assign low allocation weights to illiquid assets.
- Can we create incentives for banks to operate more closely to a liquidity-based strategy?
- Newly imposed Basel III liquidity requirements feature such a mechanism: liquidity coverage ratio.
Conclusions

- Empirical evidence suggests that banks tend to actively manage their leverage ratio.

- **Key outcome:** leverage tracking behavior may be destabilizing, causing asset prices to become highly correlated, and sensitive to shocks.

- In normal times, the downward-sloping demand curve of the nonbanking sector exerts a stabilizing force on asset prices.

- After sustained periods of growth in the banking sector, this force may no longer be sufficient to keep prices stable.

- Support policies creating incentives for strategies with higher exposure to liquid, rather than illiquid, assets.
Conclusions

Reference