Price contagion through balance sheet linkages

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Talk Outline

1. Introduction
2. The Model
3. Asset prices
4. Systemicness of leverage targeting
5. The network of asset prices
6. Policy implications and concluding remarks

Systemic Risk

- Systemically important institutions can contaminate others and spiral into shocks destabilizing the financial system.
- Current literature has put forward two main approaches for modeling systemic risk:
  - Bottom-up approach:
    - Model linkages arising when financial entities are connected via direct bilateral exposures
    - Often used towards descriptive characteristics of system via simulation with little analytical results
  - Top-down approach:
    - Global indicators of systemic risk and contribution of each financial institution
    - Granularity of system not as fine compared to bottom-up, but generally leads to more analytically tractable predictions

Top-down approach

- Spread of distress across entities captured by systemic risk measures.
- Noticeable contributions include:
  - Adrian and Brunnermeier (2001): CoVar relates systemic risk contribution of an entity to value at risk of the overall system
  - Acharaya et al. (2012): systemic expected shortfall index to measure expected amount of undercapitalization under systemic events
  - Brownless and Engle (2014): SRISK to measure expected capital shortfall under prolonged period of market distress

Balance sheet contagion vs Network Models

- Network studies assume asset prices fixed at their book values: balance sheets only take hits at default events.
- Empirical evidence suggests that financial institutions react to asset price changes by actively managing their balance sheets.
- Distress propagation: forced sales of illiquid assets may depress prices, and prompt financial distress at other banks with similar holdings.
- Adrian and Shin (2008): *if the domino model of financial contagion were the relevant one, defaults on products such as subprime mortgages would have had a much smaller impact.*

Our contribution

- Top-down model aiming at quantifying price linkages arising when firms, holding similar assets on their balance sheets, manage their leverage ratios to conform with pre-specified target levels.
- Related contributions include:
  - Shleifer and Vishny (2011): how asset fire sales lead to downward spirals or cascades in asset prices
  - Brunnermier and Pedersen (2009): interaction of market and funding liquidity
  - Greenwood, Landier and Thesmar (2014): how distribution of bank leverage and risk exposures lead to formation of systemic risk
  - Cont et al (2015): fire sales and amplification effects arising when banks need to maintain minimal leverage requirements

Motivating Evidence

- Empirical studies indicate that banks actively manage leverage
  - Gropp and Heider (2010) find that banks adjust toward their target leverages at fast speeds
  - Berger et al. (2008) find that poorly capitalized banks adjust toward their targets more quickly than well capitalized banks
- Adrian and Shin (2008, 2010) find that commercial banks track their leverage ratios, while investment banks even have procyclical leverage
- Adrian and Shin (2010): micro-foundation of leverage procyclicality driven by value at risk

Adrian and Shin (2010)

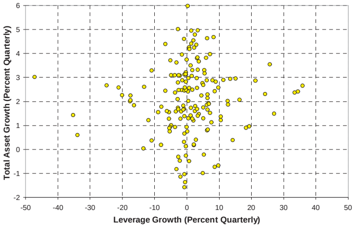


Figure 2.3: Total Assets and Leverage of Commercial Banks

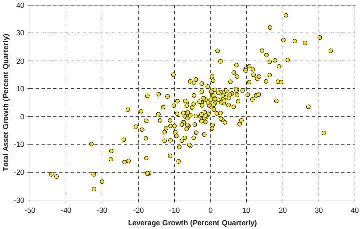


Figure 2.4: Total Assets and Leverage of Security Brokers and Dealers

Market Participants and Assets

- Market divided into two sectors:
  - Banking: commercial and investment banks.
  - Non-banking: mutual, money market and pension funds, insurances.
- Banks manage their leverage ratios to conform with a pre-specified target level
- Non-banking sector is not subject to stringent leverage management

Balance sheet management

- State of each bank described by its balance sheet which consists of assets, equity and debt.
- Banks manage their capital structure by buying and selling assets, and by increasing or reducing their level of debt.
- Banks do not raise new equity capital in response to a positive or negative shock to the asset value.

Adrian, Colla and Shin (2012)

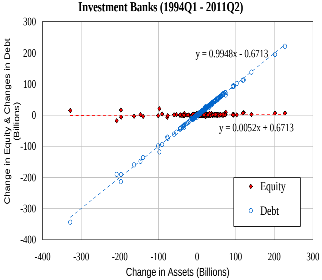


Figure 3. Scatter chart of $\{(\Delta A_t, \Delta E_t)\}$ and $\{(\Delta A_t, \Delta D_t)\}$ for changes in assets, equity and debt of US investment bank sector consisting of Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch and Morgan Stanley between Q1:1994 and Q2:2011 (Source: SEC 10Q filings).

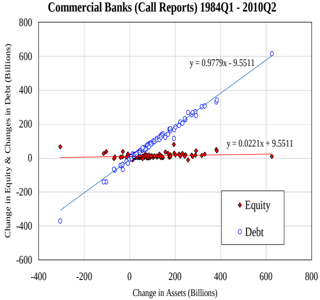


Figure 4. Scatter chart of $\{(\Delta A_t, \Delta E_t)\}$ and $\{(\Delta A_t, \Delta D_t)\}$ for changes in assets, equity and debt of US commercial bank sector at t between Q1:1984 and Q2:2010 (Source: FDIC call reports).

Assets of banking sector

- K types of assets. Market prices at t denoted by P_t^k:

$$P_t = (P_t^1 \; P_t^2 \; \cdots \; P_t^K)^\top$$

- The aggregate supply of each asset is fixed and given by

$$Q_{\mathrm{tot}} = (Q_{\mathrm{tot}}^1 \; \cdots \; Q_{\mathrm{tot}}^K)^\top$$

- N banks. Holdings of bank i are given by

$$Q_t^i = (Q_t^{1i} \; Q_t^{2i} \; \cdots \; Q_t^{Ki})^\top .$$

- Market value of the i:th bank's holdings of asset k is $A_t^{ki} = P_t^k Q_t^{ki}$. Then

$$A_t^i = (A_t^{1i} \; A_t^{2i} \; \cdots \; A_t^{Ki})^\top .$$

Debt of banking sector

- Debt of bank i at time t is D_t^i.
- Key behavioral assumption: each bank i tracks a fixed target leverage λ_i
- This yields the leverage equation:

$$\frac{D_t^i}{\mathbf{1}^\top A_t^i - D_t^i} = \lambda_i$$

- Interest rate $r = 0$ to simplify exposition

Bank's operations

- Each bank receives revenues ΔR_t^i, net of operating costs and dividends, over each time interval $[t, t+\Delta t]$
- Cash flows result from
  - (i) operating revenues
  - (ii) increase or reduction in debt used by the bank to either purchase more assets or liquidate part of its current holdings.
- Cash flow allocation strategy

$$\alpha_t^i = (\alpha_t^{1i} \ \cdots \ \alpha_t^{Ki})^\top, \qquad \alpha_t^{ki} > 0$$

with $\sum_{k=1}^K \alpha_t^{ki} = 1$

Cash flow equation

- The fundamental cash flow equation is

$$P^k_{t+\Delta t}\Delta Q^{ki}_t = \alpha^{ki}_t \left(\Delta R^i_t + \Delta D^i_t\right)$$

- LHS: amount invested by bank i in asset k at $t + \Delta t$
- RHS: proceeds from operating revenues and debt issuance in time interval $[t, t + \Delta t]$

The demand curve

- Combining leverage ratio and cash flow equation leads to

$$\Delta Q_t^{ki} = \frac{\alpha_t^{ki}}{P_{t+\Delta t}^k}\left(\lambda_i Q_t^{i\top} \Delta P_t + (1+\lambda_i)\Delta R_t^i\right)$$

- In the absence of revenue shocks and if $\Delta P_t^h = 0$ for $h \neq k$

$$\frac{\Delta Q_t^{ki}}{Q_t^{ki}} = \lambda_i \alpha_t^{ki} \frac{\Delta P_t^k}{P_{t+\Delta t}^k}$$

Upward sloping demand

$$\frac{\Delta Q_t^{ki}}{Q_t^{ki}} = \lambda_i \alpha_t^{ki} \frac{\Delta P_t^k}{P_{t+\Delta t}^k}$$

- $\lambda_i \alpha_t^{ki}$ can be interpreted as price elasticity.
  - Suppose the price of one unit of the k-th asset rises from t to $t + \Delta t$.
  - The firm increases its debt level to track leverage ratio,
  - The firm invests the raised capital by purchasing asset units.

Non-banking demand

- Non-bank institutions also trade in the available assets
- Non-banking sector holds a quantity $Q_t^{k,\mathrm{nb}}$ of asset k at t, $A_t^{k,\mathrm{nb}} = P_t^k Q_t^{k,\mathrm{nb}}$
- The incremental demand is given by

$$\Delta Q_t^{k,\mathrm{nb}} = \frac{\gamma_k}{P_{t+\Delta t}^k} Q_t^{k,\mathrm{nb}} \left(\Delta Z_t^k - \Delta P_t^k \right)$$

where ΔZ_t^k are asset-specific demand shocks

Downward-sloping demand

- Assume no leverage ratio trackers. Market clearing leads to asset price dynamics $\Delta P_t^k = \Delta Z_t^k$.
- Relationship between price and demand conditional on $\Delta Z_t^k = 0$ given by

$$\frac{\Delta Q_t^{k,\mathrm{nb}}}{Q_t^{k,\mathrm{nb}}} = -\gamma_k \frac{\Delta P_t^k}{P_{t+\Delta t}^k}$$

- $\gamma_k > 0$: elasticity of nonbanking demand for asset k

Notations and Definitions

- Given vectors $u = (u_1 \ \cdots \ u_n)^\top$ and $v = (v_1 \ \cdots \ v_n)^\top$
  - The componentwise product is

$$u \circ v = (u_1 v_1 \ \cdots \ u_n v_n)^\top$$

  - The componentwise ratio is

$$\frac{u}{v} = \left(\frac{u_1}{v_1} \ \cdots \ \frac{u_n}{v_n}\right)^\top$$

  - $\mathrm{Diag}(u)$ is the diagonal matrix with u on the diagonal.

Systemicness matrix

- The systemicness matrix $\boldsymbol{S}_t$ is given by

$$\boldsymbol{S}_t = \sum_{i=1}^{N} \frac{\alpha_t^i}{\gamma \circ A_t^{\mathrm{nb}}} \lambda_i A_t^{i\top}$$

- Its componentwise form is

$$S_t^{k\ell} = \sum_{i=1}^{N} \alpha_t^{ki} \frac{\lambda_i A_t^{\ell i}}{\gamma_k A_t^{k,\,\mathrm{nb}}}$$

- Key determinant for excess price correlation induced by the leverage targeting banks.

Graph based interpretation

- Systemicness matrix $S_t^{k\ell} = \sum_{i=1}^{N} \alpha_t^{ki} \frac{\lambda_i A_t^{\ell i}}{\gamma_k A_t^{k,\text{nb}}}$ interpreted as weighted adjacency matrix of network:
  - Directed edge from node ℓ to node k with weight $S_t^{k\ell}$.
  - Return shock to asset ℓ of size y_ℓ propagates along the edge (ℓ, k) and results in a shock to asset k of size $y_k = S_t^{k\ell} y_\ell$
  - The shock y_k forces further leverage adjustments, causing return shocks to other assets, and so on.

Market dynamics

Proposition

The cash flow equation, leverage equation, the non-banking demand function, and market clearing imply

$$\frac{\Delta P_t}{P_t} = (\boldsymbol{I} - \boldsymbol{S}_t)^{-1} \underbrace{\left[\frac{\Delta Z_t}{P_t} + \sum_{i=1}^{N} \frac{\alpha_t^i}{\gamma \circ A_t^{\mathrm{nb}}} (1+\lambda_i) \Delta R_t^i \right]}_{\text{vector of initial aggregate return shocks}}$$

$$\Delta A_t^i = Q_t^i \circ \Delta P_t + \alpha_t^i \left(\lambda_i Q_t^{i\top} \Delta P_t + (1+\lambda_i) \Delta R_t^i \right)$$

$$\Delta A_t^{\mathrm{nb}} = Q_t^{\mathrm{nb}} \circ \left(\gamma \circ \Delta Z_t - (\gamma - \mathbf{1}) \circ \Delta P_t \right)$$

assuming that the matrix inverse exists.

Systemic Risk

- Suppose all eigenvalues of $\boldsymbol{S}_t$ are less than one. Then

$$\frac{\Delta P_t}{P_t} = \left[\boldsymbol{I} + \boldsymbol{S}_t + \boldsymbol{S}_t^2 + \boldsymbol{S}_t^3 + \ldots\right] \Delta Y_t$$

- Direct impact: first term $\boldsymbol{I}$ in the power series expansion
- Indirect effect: Suppose negative shock occurs.
  - Term $\boldsymbol{S}_t$ corresponds to first round of deleveraging.
  - Term $\boldsymbol{S}_t^2$ corresponds to a second round of deleveraging.
  - Each round impacts prices and total realized return is the aggregate outcome of this process

The spectral radius of the systemicness matrix

- The spectral radius $\rho(\boldsymbol{S}_t)$, $\boldsymbol{S}_t = \sum_{i=1}^{N} \frac{\alpha_t^i}{\gamma \circ A_t^{\text{nb}}} \lambda_i A_t^{i\top}$, is the aggregate level of vulnerability in the system

Proposition

Assume all asset holdings and all cash flow allocation weights α_t^{ki} are nonnegative. We have the bounds

$$\max_{k=1,\ldots,K} \frac{\sum_{i=1}^{N} \lambda_i \alpha_t^{ki} A_t^{ki}}{\gamma_k A_t^{k,\,\text{nb}}} \leq \rho(\boldsymbol{S}_t) \leq \max_{k=1,\ldots,K} \frac{\sum_{i=1}^{N} \lambda_i \alpha_t^{ki} \mathbf{1}^\top A_t^{i}}{\gamma_k A_t^{k,\,\text{nb}}}$$

Upper bound of spectral radius

- The upper bound is given by

$$\max_{k=1,\ldots,K} \frac{\sum_{i=1}^{N} \lambda_i \alpha_t^{ki} \mathbf{1}^\top A_t^i}{\gamma_k A_t^{k,\,\mathrm{nb}}}$$

- Quantifies the size of leverage targeting banks relative to the size of the nonbanking sector, as measured by elasticity-weighted assets.
- When nonbanking sector is large, $\rho(\boldsymbol{S}_t)$ is small, and the impact on realized returns is moderate.

Lower bound of spectral radius

- The lower bound is given by

$$\max_{k=1,\dots,K} \frac{\sum_{i=1}^{N} \lambda_i \alpha_t^{ki} A_t^{ki}}{\gamma_k A_t^{k,\,\mathrm{nb}}}$$

- Numerator only involves the banks' holdings of the k:th asset
- If the nonbanking sector is small in relative terms, $\rho(\boldsymbol{S}_t)$ is large, causing strong impact on realized returns

Fixed relative exposure strategy

- Relative exposure strategy is given by $\alpha_t^{ki} = A_t^{ki}/(\mathbf{1}^\top A_t^i)$.
- Upper bound on the spectral radius given by

$$\rho(\boldsymbol{S}_t) \leq \max_{k=1,\dots,K} \frac{\sum_{i=1}^N \lambda_i A_t^{ki}}{\gamma_k A_t^{k,\,\mathrm{nb}}}$$

- Existence of an asset class for which holdings of highly levered banks are large relatively to the nonbanking sector destabilizes system.
- Relation to illiquidity concentration measure by Duarte and Eisenbach (2013).

Liquidity based strategy I

- Banks sell (buy) the most liquid assets when there is a need to delever (lever up)
- $\alpha_t^{ki} = \frac{\gamma_k A_t^{k,\,\mathrm{nb}}}{\sum_{\ell=1}^{K} \gamma_\ell A_t^{\ell,\,\mathrm{nb}}}$
- Spectral radius equals its upper bound, i.e.

$$\rho(\boldsymbol{S}_t) = \frac{\sum_{i=1}^{N} \lambda_i \mathbf{1}^\top A_t^i}{\sum_{k=1}^{K} \gamma_k A_t^{k,\,\mathrm{nb}}}$$

Liquidity based strategy II

- Spectral radius only depends on the relative *aggregate* size of the banking and the nonbanking sectors.
- Existence of an asset class for which price elasticity is high and for which the nonbanking sector is large relative to the banking sector, would be enough to stabilize the system
- Severe deleveraging needs can be absorbed by the nonbanking sector. The remaining, potentially illiquid, assets are not subjected to fire sales.

Liquidity based strategy III

Proposition

Assume all asset holdings are nonnegative. Then

$$\underbrace{\frac{\sum_{i=1}^{N}\lambda_i \mathbf{1}^\top A_t^i}{\sum_{k=1}^{K}\gamma_k A_t^{k,\mathrm{nb}}}}_{\textit{liquidity based}} \leq \underbrace{\max_{k=1,\ldots,K}\frac{\sum_{i=1}^{N}\lambda_i A_t^{ki}}{\gamma_k A_t^{k,\mathrm{nb}}}}_{\textit{fixed exposure}}$$

One-step impact of shocks

- Deleveraging activities of bank i contributes to one-step propagation of shocks from asset ℓ to asset k:

$$\alpha_t^{ki} \frac{\lambda_i A_t^{\ell i}}{\gamma_k A_t^{k,\,\mathrm{nb}}}$$

- Total one-step return impact caused by bank i:

$$\lambda_i \mathbf{1}^\top A_t^i \sum_{k=1}^{K} \alpha_t^{ki} \frac{1}{\gamma_k A_t^{k,\,\mathrm{nb}}}$$

- Strongest one-step systemic impact by large banks that are highly levered and allocate their cash flows to illiquid assets

Aggregate impact of shocks

- One-step return impacts give an incomplete view of the systemic properties of the banking sector.
- Aggregate impact on return of asset k due to a shock in asset ℓ given by

$$(\boldsymbol{I} - \boldsymbol{S}_t)^{-1} = \boldsymbol{I} + \boldsymbol{S}_t + \boldsymbol{S}_t^2 + \cdots$$

- Can higher-order effects beyond the one-step impact be neglected?

Illustrative Example I

- Three assets ($K = 3$) and two banks ($N = 2$).
- Both banks track the same debt-over-equity leverage ratio $\lambda = 9$.
- Current assets and fixed relative exposure strategies given by

Banking Sector	Non-Banking Sector
$A_t^1 = \begin{bmatrix} 1 \\ 9 \\ 0 \end{bmatrix}$ $\alpha_t^1 = \begin{bmatrix} 0.1 \\ 0.9 \\ 0 \end{bmatrix}$ $A_t^2 = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$ $\alpha_t^2 = \begin{bmatrix} 0 \\ 0.2 \\ 0.8 \end{bmatrix}$	$\gamma \circ A_t^{\mathrm{nb}} = \begin{bmatrix} 900 \\ 400 \\ 300 \end{bmatrix}$

Illustrative Example II

- Systemicness matrix becomes

$$\boldsymbol{S}_t = \begin{bmatrix} 0.001 & 0.009 & 0 \\ 0.02 & 0.187 & 0.018 \\ 0 & 0.24 & 0.96 \end{bmatrix}, \qquad \rho(\boldsymbol{S}_t) = 0.97$$

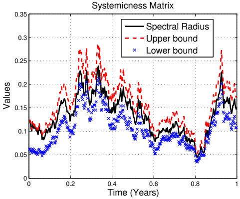
- $\boldsymbol{S}_t^{3,1} = 0$, but the $(3, 1)$ entry of $(\boldsymbol{I} - \boldsymbol{S}_t)^{-1}$ is 0.17.
- A shock to asset 1 has a sizable impact on the return of asset 3, even though the one-step effect is zero!

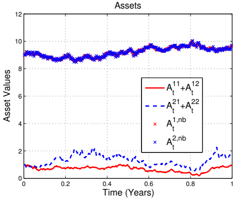
Numerical setup

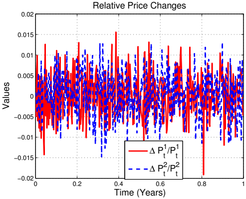
- $\Delta R_t^i \sim \mathcal{N}(0, 0.1)$, $\Delta Z_t^k \sim \mathcal{N}(0, 0.1)$
- Initial aggregate size of banking sector is 8% of the nonbanking sector
- $N = K = 2$ and $P_0^1 = P_0^2 = 1$.
- $\lambda_1 = \lambda_2 = 10$ and $\gamma_1 = \gamma_2 = 9$

Stable realization

- Fixed relative exposure strategy

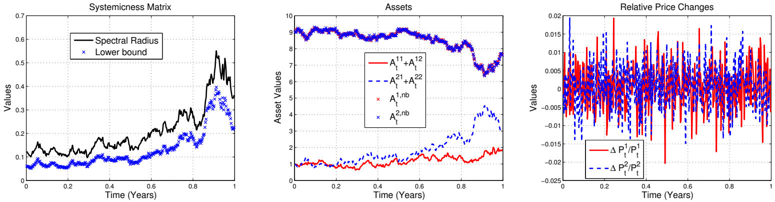




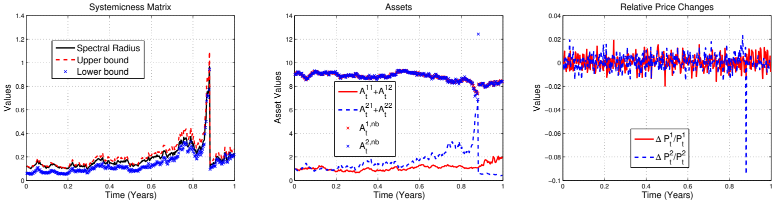


Unstable realization

- Liquidity based strategy



- Fixed relative exposure strategy



Policies suggestions?

- Spectral radius of systemicness matrix characterizes stability of price dynamics in presence of leverage targeting banks
- Stability improves if (i) the banking sector is small, (ii) the nonbanking sector is large, (iii) nonbanking demand is highly elastic, (iv) target leverage is low, or (v) banks assign low allocation weights to illiquid assets.
- Can we create incentives for banks to operate more closely to a liquidity-based strategy?
- Newly imposed Basel III liquidity requirements feature such a mechanism: liquidity coverage ratio.

Conclusions

- Empirical evidence suggests that banks tend to actively manage their leverage ratio.
- Key outcome: leverage tracking behavior may be destabilizing, causing asset prices to become highly correlated, and sensitive to shocks.
- In normal times, the downward-sloping demand curve of the nonbanking sector exerts a stabilizing force on asset prices.
- After sustained periods of growth in the banking sector, this force may no longer be sufficient to keep prices stable.
- Support policies creating incentives for strategies with higher exposure to liquid, rather than illiquid, assets.

Reference

- Agostino Capponi and Martin Larsson. Price contagion through balance sheet linkages. *Review of Asset Pricing Studies*, Forthcoming.