A Bayesian methodology for systemic risk assessment in financial networks

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Joint work with Luitgard Veraart (London School of Economics)

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The problem

- Consider interbank market as network:
  - Nodes consist of \( n \) banks with indices in \( \mathcal{N} = \{1, \ldots, n\} \).
  - Edges \( L_{ij} \) represent nominal interbank liability of bank \( i \) to bank \( j \).

- Stress tests: Suppose some banks default on their liabilities. How do losses spread along the edges? What if edges are not observable?

- A matrix \( L = (L_{ij}) \in \mathbb{R}^{n \times n} \) is a liabilities matrix if \( L_{ij} \geq 0, L_{ii} = 0 \ \forall i, j \)

- Total nominal interbank liabilities of bank \( i \): \( r_i(L) := \sum_{j=1}^{m} L_{ij} \).

- Total nominal interbank assets of bank \( i \): \( c_i(L) := \sum_{j=1}^{m} L_{ji} \).

- In practice, \( L_{ij} \) not fully observable, but \( r_i(L), c_i(L) \) are.

- How to fill in the missing data? Implications for stress testing?
Previous Approaches

1. **Entropy method** (Upper & Worms, 2004).
   - Minimise the Kullback-Leibler divergence between $L$ and a specified input matrix, subject to the linear constraints.
   - Widely used (e.g. interbank exposures for Germany (Upper & Worms, 2004), UK (Wells, 2004; Elsinger et al., 2006) Belgium (Degryse & Nguyen, 2007)).
   - Resulting network usually complete, i.e., all entries of $L$ (except on diagonal) are non-zero. *Only point estimate.*

2. **Minimum density (MD) method** (Anand et al., 2014):
   - Minimises the total number of edges consistent with the aggregated interbank assets and liabilities.
   - *Only a point estimate.*

3. **Simulation-based approach** (Hałaj & Kok, 2013).
   - Mechanism to randomly generate different network structures consistent with observed aggregates.
   - Probabilistic model not completely characterised.
Main contributions

▶ Bayesian model for liabilities matrix. Interested in the distribution of liabilities matrix conditional on its row and column sums and conditional on some other observed elements of $L$.

▶ MCMC method to generate samples (Gibbs sampler).

▶ Application to systemic risk assessment: Gives probabilities for outcomes of stress tests.

▶ Code is available as R-package (systemicrisk) on CRAN.
Existence of admissible liabilities matrix

Some elements of $L$ may be known; given by $L^* \in \mathcal{L}^* := (\{*\} \cup [0, \infty))^{n \times n}$ where $L^*_{ij} = *$ means that the liability between $i$ and $j$ is unknown.

**Theorem**

Consider $a \in [0, \infty)^n$, $l \in [0, \infty)^n$ and $L^* \in \mathcal{L}^*$ satisfying $r(L^*) \leq l$, $c(L^*) \leq a$ and $\sum_{i=1}^n a_i = \sum_{i=1}^n l_i$. Then the following are equivalent:

1. There exists an admissible liabilities matrix $L$ for $a$ and $l$ respecting $L^*$.

2. $\forall I \subset \mathcal{N}, J \subset \mathcal{N}$ with $L^*_{ij} \neq * \forall i \in I, j \in J$ we have

   $$\sum_{i \in I} \tilde{l}_i + \sum_{j \in J} \tilde{a}_j \leq A$$

   where $\tilde{l} = l - r(L^*)$ and $\tilde{a} = a - c(L^*)$ and $A = \sum_{i=1}^n \tilde{l}_i$.

Proof: problem is equivalent to a maximum flow problem (efficient algorithms for constructing solution).
The Basic model

- Constructs adjacency matrix $\mathcal{A} = (A_{ij})$; attaches liabilities $L_{ij}$.
- Model:

$$
P(A_{ij} = 1) = p_{ij},
L_{ij} | \{A_{ij} = 1\} \sim \text{Exponential}(\lambda_{ij}).
$$

- Parameters:
  - $p \in [0, 1]^{n \times n}$, $p_{ij}$ probability of existence of directed edge from $i$ to $j$, often: $\text{diag}(p) = 0$;
  - $\lambda \in \mathbb{R}^{n \times n}$, governs distribution of weights given that edge exists.
### The Basic model

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- **Model:**

  
  $P(A_{ij} = 1) = p_{ij},$
  
  $L_{ij}|\{A_{ij} = 1\} \sim \text{Exponential}(\lambda_{ij}).$ (2)

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- **Observations:**

  $r(L) = l, \quad c(L) = a, \quad L \equiv L^*,$
The Basic model

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- **Model:**

  $$
  \mathbb{P}(A_{ij} = 1) = p_{ij},
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  $$

- **Parameters:**
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  - $\lambda \in \mathbb{R}^{n \times n}$, governs distribution of weights given that edge exists.

- **Observations:** $r(L) = l$, $c(L) = a$, $L \equiv L^*$,

- **Main interest:** Distribution of $h(L) \mid a, l$. 
(Unconditional) distribution of out degrees in an example

\[ n = 100, \ p_{ij} = 0.3 \mathbb{I}(i \neq j) \]
Gibbs sampling for $L \mid a, l$

- Markov Chain Monte Carlo (MCMC): Interested in sampling from a given distribution. Construct a Markov chain with this stationary distribution. Run chain. Chain converges to stationary distribution.

- Key idea of Gibbs sampler: a step of the chain updates one or several components of the entire parameter vector by sampling them from their joint conditional distribution given the remainder of the parameter vector.

- Here parameter vector is matrix $L$:
  - Initialise chain with matrix $L$ that satisfies $r(L) = l$, $c(L) = a$.
  - MCMC sampler produce a sequence of matrices $L^1, L^2, \ldots$.

- Quantity of interest: $\mathbb{E}[h(L) \mid l, a] \approx \frac{1}{N} \sum_{i=1}^{N} h(L^{i\delta+b})$

  $N$ number of samples, $b$ burn-in period, $\delta \in \mathbb{N}$ thinning parameter.
Updating components of $L$

- Need to decide which elements of $L$ need to be updated.

- Need to determine how the new values will be chosen, i.e., need to determine their distribution conditional on remainder of elements of $L$. 
Illustration of updating submatrices

\[
\begin{array}{ccc}
L_{i_1 j_1} & L_{i_1 j_2} & \text{ } \\
L_{i_2 j_1} & L_{i_2 j_2} & L_{i_2 j_3} \\
L_{i_3 j_1} & L_{i_3 j_2} & L_{i_3 j_3} \\
L_{i_4 j_1} & L_{i_4 j_2} & \text{ } \\
\end{array}
\]
### A Bayesian Approach to Systemic Risk

**Updating - Illustration**

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A Gandy
Balance sheets and fundamental defaults

▶ Balance sheet of bank $i$:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tbody>
<tr>
<td>external assets</td>
<td>external liabilities</td>
</tr>
<tr>
<td>interbank assets</td>
<td>net worth</td>
</tr>
<tr>
<td>$a_i^{(e)}$</td>
<td>$l_i^{(e)}$</td>
</tr>
<tr>
<td>$a_i := c_i(L)$</td>
<td>$l_i := r_i(L)$</td>
</tr>
</tbody>
</table>

▶ Stress tests: apply proportional shock $s \in [0, 1]^n$ to external assets; shocked external assets are $s_i a_i^{(e)} \forall i$.

▶ Fundamental defaults: $\{ i \mid w_i(L, s_i a_i^{(e)}, l_i^{(e)}) < 0 \}$

▶ Fundamental defaults can be checked from balance sheet aggregates without needing to know the whole matrix $L$!

▶ To check for contagious defaults we need to know $L$. 
Empirical example - data

Balance sheet data (in million Euros) from banks in the EBA 2011 stress test:

<table>
<thead>
<tr>
<th>Bank code</th>
<th>Bank</th>
<th>(a^{(e)} + a)</th>
<th>(a)</th>
<th>(w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE017</td>
<td>DEUTSCHE BANK AG</td>
<td>1,905,630</td>
<td>47,102</td>
<td>30,361</td>
</tr>
<tr>
<td>DE018</td>
<td>COMMERZBANK AG</td>
<td>771,201</td>
<td>49,871</td>
<td>26,728</td>
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<tr>
<td>DE019</td>
<td>LANDES_BANK BADEN-WURTTEMBER</td>
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<tr>
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<td>DZ BANK AG</td>
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<td>100,099</td>
<td>7,299</td>
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<td>BAYERISCHE LANDES BANK</td>
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<td>66,535</td>
<td>11,501</td>
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<tr>
<td>DE022</td>
<td>NORDDEUTSCHE LANDES BANK -GZ-</td>
<td>228,586</td>
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<tr>
<td>DE024</td>
<td>WESTLB AG, DUSSELDORF</td>
<td>191,523</td>
<td>24,007</td>
<td>4,218</td>
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<tr>
<td>DE025</td>
<td>HSH NORD BANK AG, HAMBURG</td>
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<td>4,645</td>
<td>4,434</td>
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<td>DEKABANK DEUTSCHE GIROZENTRALE</td>
<td>130,304</td>
<td>30,937</td>
<td>3,359</td>
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Stress testing

- **Deterministic shock to external assets** of all 11 banks in the network by reducing external assets by 3%.

- Shock causes **fundamental default of 4 banks**: DE017, DE022, DE023, DE024.

- We apply the **clearing approach** by Eisenberg & Noe (2001) and [Rogers & V. (2013)] to determine which banks suffer **contagious defaults**.

- Gibbs sampler allows us to derive posteriori **default probabilities** for remaining 7 banks.
Default probabilities of banks as a function of $p$
Default probabilities for clearing with default costs

(a) Clearing with $\alpha = 1, \beta = 0.7$
Mean out-degree of banks, i.e., $\mathbb{E}\left[\sum_j A_{ij} \mid a, l\right]$, for different $p_{\text{ER}}$ in the Erdős-Rényi network

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Hierarchical Models

- The basic model can be used as a building block in more complicated models, e.g. in hierarchical models:

\[
\theta \sim \pi(\theta),
\]

\[
(p_{ij}, \lambda_{ij})_{i,j \in \mathcal{N}} = f(\theta),
\]  \hspace{1cm} (3)

where \( \pi \) is an a-priori distribution on \( \theta \) and \( f \) is a given function.

- Sampling of \( \theta, L|l, a, L^* \) combined by iterating between sampling

- \( L|\theta, l, a, L^* \) (using the Gibbs sampler)

- and \( \theta|L \) (using more standard MCMC techniques, e.g. Gibbs, Metropolis Hasting).
Example: Conjugate distribution model

$p$ and $\lambda$ consist of identical but random values; 
$\theta = (\tilde{p}, \tilde{\lambda})$

$$\tilde{p} \sim \text{Beta}(a, b), \quad \tilde{\lambda} \sim \text{Gamma}(c, d)$$

$$p_{ij} = \tilde{p} \mathbb{1}(i \neq j), \quad \lambda_{ij} = \tilde{\lambda}, \quad i, j \in \mathcal{N}.$$  

for some parameters $a, b, c, d$.

- prior on $\tilde{p}$, $\tilde{\lambda}$ is flexible,

- direct sampling of $\theta|L$ possible (conjugate distributions).

- Extensions: Partition $p, \lambda$, use independent models for partitions.
Fitness model - Power law in Degree Distr & Weights

- Empirical literature (Boss et al., 2004, e.g.) suggests that power laws are reasonable models for degree distributions AND for liabilities, i.e. their density would be of the form

\[ p(x) = cx^\alpha \]

Empirical studies often find \( \alpha \) between \(-2\) and \(-3\).

- Servedio et al. (2004): fitness-based model for degree distributions.

- We will couple such a fitness model for the degrees with a model for liabilities that allows a power law. Key ideas
  - Use same fitness for both liabilities and degrees.
  - Gamma mixture of exponential distributions lead to a Pareto type II distribution (also called Lomax distribution).
Fitness model - Power law in Degree Distr & Weights

\[ X_i \sim \text{Exp}(1), \ i \in \mathcal{N}, \]
\[ p_{ij} = f(X_i + X_j)\mathbb{I}(i \neq j), \ i, j \in \mathcal{N}, \]
\[ \lambda_{ij} = G^{-1}_{\zeta,\eta}(\exp(-X_i)) + G^{-1}_{\zeta,\eta}(\exp(-X_j)), \ i, j \in \mathcal{N} \]
\[ (\zeta, \eta) \sim \pi(\zeta, \eta), \]

where \( G^{-1}_{\zeta,\eta} \) is the quantile function of a Gamma distr. with parameter \( \zeta > 0, \eta > 0 \). \( \pi \) is a prior distribution on \( (\zeta, \eta) \) and

\[
f(x) := \begin{cases} 
\beta \left( \frac{\gamma}{\beta} \right)^{1-\exp(-x)} \left( 1 - \log \left( \frac{\gamma}{\beta} \right) \exp(-x) \right), & \text{if } \alpha = -1, \\
\beta \left( \xi + (1 - \xi) e^{-x} \right) \frac{1}{\alpha+1} \left\{ 1 + \frac{1}{\alpha+1} \frac{1-\xi}{\xi e^x + 1-\xi} \right\}, & \text{if } \alpha \neq -1,
\end{cases}
\]

where \( \xi := \left( \frac{\gamma}{\beta} \right)^{\alpha+1} \).
Realisations of Out Degree Distributions

\[ n = 100; \alpha = -2.5; \beta = 0.2; \gamma = 1.0 \]

Theoretical dist \( \approx \) distr of \( \sum_{j=1}^{n} p_{ij} \)
Flexibility in the degree distribution (different choices of $\alpha$, $\beta$, $\gamma$).
Log-log plot of the survival function of $L$

Black: theoretical distribution (type II Pareto)
Gray: Realisations

$P(L_{ij} \geq x)$
Mean out-degree of banks, i.e., $\mathbb{E}[\sum_j A_{ij} \mid a, l]$

<table>
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<th>ER $p_{ER}$</th>
<th>Fitness $\alpha, \beta, \gamma$</th>
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Summary

- Bayesian setup can be used to fill in “missing” information in a principled way. Large flexibility.

- Construction of Gibbs sampler for sampling from distribution of liabilities matrix conditional on its row and column sums. R package (systemicrisk) available from CRAN. Some theoretical results + simulation studies show that sampler works.

- Can be used for stress tests using empirical data.

- Can be extended to incorporate additional information such as expert views etc. on the network structure:
  - Hierarchical model for $p, \Lambda \ (\rightarrow \text{power laws})$.
  - Observation of some components of the matrix.
References I


References II


Identifiability

The matrices $p$ and $\lambda$ cannot be fully identified merely by observing the row and column sums.

Lemma

Let $l, a \in (0, \infty)^n$ with $A = \sum_{i=1}^{n} l_i = \sum_{i=1}^{n} a_i$ and $a_i + l_i < A$ for all $i$. Let $p \in [0, 1]^{n \times n}$ with $p_{ij} > 0 \ \forall i \neq j$, $\text{diag}(p) = 0$. Then

$$\exists (\lambda_{ij}) : \forall j : \sum_{i=1}^{n} E(L_{ij}) = a_j \text{ and } \sum_{j=1}^{n} E(L_{ij}) = l_j.$$  

Hence: need to make assumptions about $p$. 