# A Bayesian methodology for systemic risk assessment in financial networks

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# The problem

- Consider interbank market as network:
  - Nodes consist of $n$ banks with indices in $\mathcal{N} = \{1, \dots, n\}$.
  - Edges $L_{ij}$ represent nominal interbank liability of bank $i$ to bank $j$.
- Stress tests: Suppose some banks default on their liabilities. How do losses spread along the edges? What if edges are not observable?
- A matrix $L = (L_{ij}) \in \mathbb{R}^{n \times n}$ is a liabilities matrix if $L_{ij} \geq 0$, $L_{ii} = 0\ \forall i, j$
- Total nominal interbank liabilities of bank $i$: $r_i(L) := \sum_{j=1}^{m} L_{ij}$.
- Total nominal interbank assets of bank $i$: $c_i(L) := \sum_{j=1}^{m} L_{ji}$.
- In practice, $L_{ij}$ not fully observable, but $r_i(L)$, $c_i(L)$ are.
- How to fill in the missing data? Implications for stress testing?

# Previous Approaches

1. Entropy method (Upper & Worms, 2004).
   - Minimise the Kullback-Leibler divergence between $L$ and a specified input matrix, subject to the linear constraints.
   - Widely used (e.g. interbank exposures for Germany (Upper & Worms, 2004), UK (Wells, 2004; Elsinger et al., 2006) Belgium (Degryse & Nguyen, 2007)).
   - Resulting network usually complete, i.e., all entries of $L$ (except on diagonal) are non-zero. Only point estimate.
2. Minimum density (MD) method (Anand et al., 2014):
   - Minimises the total number of edges consistent with the aggregated interbank assets and liabilities.
   - Only a point estimate.
3. Simulation-based approach (Hałaj & Kok, 2013).
   - Mechanism to randomly generate different network structures consistent with observed aggregates.
   - Probabilistic model not completely characterised.

# Main contributions

- Bayesian model for liabilities matrix. Interested in the distribution of liabilities matrix conditional on its row and column sums and conditional on some other observed elements of $L$.

- MCMC method to generate samples (Gibbs sampler).

- Application to systemic risk assessment: Gives probabilities for outcomes of stress tests.

- Code is available as R-package (systemicrisk) on CRAN.

# Existence of admissible liabilities matrix

Some elements of $L$ may be known; given by $L^* \in \mathcal{L}^* := (\{*\} \cup [0,\infty))^{n\times n}$ where $L^*_{ij} = *$ means that the liability between $i$ and $j$ is unknown.

**Theorem**

*Consider $a \in [0,\infty)^n, l \in [0,\infty)^n$ and $L^* \in \mathcal{L}^*$ satisfying $r(L^*) \leq l$, $c(L^*) \leq a$ and $\sum_{i=1}^n a_i = \sum_{i=1}^n l_i$. Then the following are equivalent:*

1. *There exists an admissible liabilities matrix $L$ for $a$ and $l$ respecting $L^*$.*
2. *$\forall I \subset \mathcal{N}, J \subset \mathcal{N}$ with $L^*_{ij} \neq * \forall i \in I, j \in J$ we have*

$$\sum_{i\in I} \tilde{l}_i + \sum_{j \in J} \tilde{a}_j \leq A \tag{1}$$

*where $\tilde{l} = l - r(L^*)$ and $\tilde{a} = a - c(L^*)$ and $A = \sum_{i=1}^n \tilde{l}_i$.*

Proof: problem is equivalent to a maximum flow problem (efficient algorithms for constructing solution).

# The Basic model

- Constructs adjacency matrix $\mathcal{A} = (\mathcal{A}_{ij})$; attaches liabilities $L_{ij}$.
- Model:

$$\begin{aligned} \mathbb{P}(\mathcal{A}_{ij} = 1) &= p_{ij}, \\ L_{ij} | \{\mathcal{A}_{ij} = 1\} &\sim \text{Exponential}(\lambda_{ij}). \end{aligned} \tag{2}$$

- Parameters:
  - $p \in [0, 1]^{n \times n}$, $p_{ij}$ probability of existence of directed edge from $i$ to $j$, often: $\text{diag}(p) = 0$;
  - $\lambda \in \mathbb{R}^{n \times n}$, governs distribution of weights given that edge exists.

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- Observations: $r(L) = l, \quad c(L) = a, \quad L \equiv L^*,$

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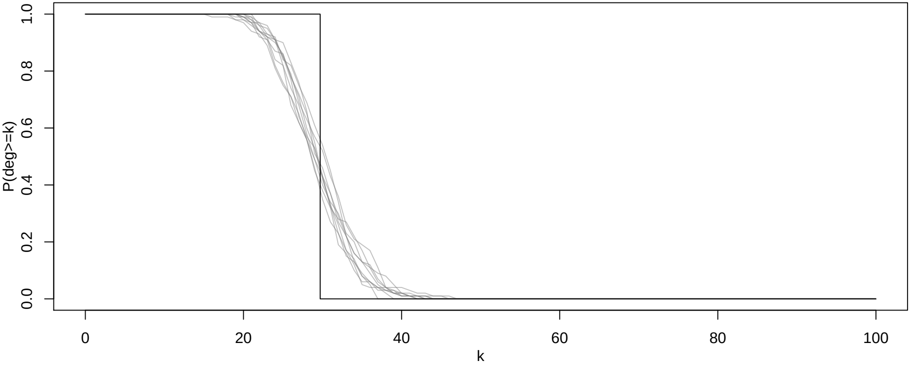
  - $\lambda \in \mathbb{R}^{n \times n}$, governs distribution of weights given that edge exists.
- Observations: $r(L) = l, \quad c(L) = a, \quad L \equiv L^*,$
- Main interest: Distribution of $h(L) \mid a, l$.

# (Unconditional) distribution of out degrees in an example

$n = 100$, $p_{ij} = 0.3\mathbb{I}(i \neq j)$



# Gibbs sampling for $L|a, l$

- Markov Chain Monte Carlo (MCMC): Interested in sampling from a given distribution. Construct a Markov chain with this stationary distribution. Run chain. Chain converges to stationary distribution.
- Key idea of Gibbs sampler: a step of the chain updates one or several components of the entire parameter vector by sampling them from their joint conditional distribution given the remainder of the parameter vector.
- Here parameter vector is matrix $L$:
  - Initialise chain with matrix $L$ that satisfies $r(L) = l$, $c(L) = a$.
  - MCMC sampler produce a sequence of matrices $L^1, L^2, \dots$.
  - Quantity of interest: $\mathbb{E}[h(L)|l, a] \approx \frac{1}{N} \sum_{i=1}^{N} h(L^{i\delta+b})$,

    $N$ number of samples, $b$ burn-in period, $\delta \in \mathbb{N}$ thinning parameter.

# Updating components of $L$

- Need to decide which elements of $L$ need to be updated.
- Need to determine how the new values will be chosen, i.e., need to determine their distribution conditional on remainder of elements of $L$.

# Illustration of updating submatrices

| | |
|---|---|
| $L_{i_1 j_1}$ | $L_{i_1 j_2}$ |
| $L_{i_2 j_1}$ | $L_{i_2 j_2}$ |

| | | |
|---|---|---|
| | $L_{i_1 j_1}$ | $L_{i_1 j_2}$ |
| $L_{i_2 j_3}$ | | $L_{i_2 j_2}$ |
| $L_{i_3 j_3}$ | $L_{i_3 j_1}$ | |

| | | | |
|---|---|---|---|
| | $L_{i_1 j_1}$ | $L_{i_1 j_2}$ | |
| | | $L_{i_2 j_2}$ | $L_{i_2 j_3}$ |
| $L_{i_3 j_4}$ | | | $L_{i_3 j_3}$ |
| $L_{i_4 j_4}$ | $L_{i_4 j_1}$ | | |

# Updating - Illustration

| |
|---|
| 47 |
| 49.9 |
| 91.3 |
| 99.9 |
| 66.5 |
| 54.9 |

| | | | | | |
|---|---|---|---|---|---|
| 47.1 | 49.8 | 91.2 | 100 | 66.5 | 54.9 |

→

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 14.7 | 32.3 | 0 | 0 | | 47 |
| 0 | | 19.5 | 11 | 0 | 19.5 | | 49.9 |
| 23.5 | 20.3 | | 20.5 | 20.5 | 6.4 | | 91.3 |
| 23.5 | 20.3 | 23.5 | | 23 | 9.5 | | 99.9 |
| 0 | 9.1 | 19.5 | 18.4 | | 19.5 | | 66.5 |
| 0 | 0 | 14 | 17.9 | 23 | | | 54.9 |

| | | | | | |
|---|---|---|---|---|---|
| 47.1 | 49.8 | 91.2 | 100 | 66.5 | 54.9 |

→

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | **13.9** | 32.3 | 0 | **0.8** | | 47 |
| 0 | | 19.5 | 11 | 0 | 19.5 | | 49.9 |
| 23.5 | 20.3 | | 20.5 | 20.5 | 6.4 | | 91.3 |
| 23.5 | 20.3 | 23.5 | | 23 | 9.5 | | 99.9 |
| 0 | 9.1 | **20.3** | 18.4 | | **18.6** | | 66.5 |
| 0 | 0 | 14 | 17.9 | 23 | | | 54.9 |

| | | | | | |
|---|---|---|---|---|---|
| 47.1 | 49.8 | 91.2 | 100 | 66.5 | 54.9 |

→

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 13.9 | 32.3 | 0 | 0.8 | | 47 |
| 0 | | **35.5** | 11 | 0 | **3.5** | | 49.9 |
| 23.5 | 20.3 | | **4.5** | 20.5 | **22.5** | | 91.3 |
| 23.5 | 20.3 | 23.5 | | 23 | 9.5 | | 99.9 |
| 0 | 9.1 | **4.3** | **34.4** | | 18.6 | | 66.5 |
| 0 | 0 | 14 | 17.9 | 23 | | | 54.9 |

| | | | | | |
|---|---|---|---|---|---|
| 47.1 | 49.8 | 91.2 | 100 | 66.5 | 54.9 |

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 13.9 | 32.3 | 0 | 0.8 | | 47 |
| **0** | | 35.5 | 11 | 0 | **3.5** | | 49.9 |
| 23.5 | 20.3 | | 4.5 | 20.5 | 22.5 | | 91.3 |
| **23.5** | 20.3 | 23.5 | | 23 | **9.5** | | 99.9 |
| 0 | 9.1 | 4.3 | 34.4 | | 18.6 | | 66.5 |
| 0 | 0 | 14 | 17.9 | 23 | | | 54.9 |

| | | | | | |
|---|---|---|---|---|---|
| 47.1 | 49.8 | 91.2 | 100 | 66.5 | 54.9 |

→

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 13.9 | 32.3 | 0 | 0.8 | | 47 |
| 0 | | 35.5 | 11 | 0 | 3.5 | | 49.9 |
| 23.5 | **30.9** | | 4.5 | **9.9** | 22.5 | | 91.3 |
| 23.5 | **9.8** | 23.5 | | **33.6** | 9.5 | | 99.9 |
| 0 | 9.1 | 4.3 | 34.4 | | 18.6 | | 66.5 |
| 0 | 0 | 14 | 17.9 | 23 | | | 54.9 |

| | | | | | |
|---|---|---|---|---|---|
| 47.1 | 49.8 | 91.2 | 100 | 66.5 | 54.9 |

# Balance sheets and fundamental defaults

- Balance sheet of bank $i$:

| Assets | | Liabilities | |
|---|---|---|---|
| external assets | $a_i^{(e)}$ | external liabilities | $l_i^{(e)}$ |
| interbank assets | $a_i := c_i(L)$ | interbank liabilities | $l_i := r_i(L)$ |
| | | net worth | $w_i(L, a_i^{(e)}, l_i^{(e)})$ |

- Stress tests: apply proportional shock $s \in [0,1]^n$ to external assets; shocked external assets are $s_i a_i^{(e)} \, \forall i$.

- Fundamental defaults: $\{i \mid w_i(L, s_i a_i^{(e)}, l_i^{(e)}) < 0\}$

- Fundamental defaults can be checked from balance sheet aggregates without needing to know the whole matrix $L$!

- To check for contagious defaults we need to know $L$.

# Empirical example - data

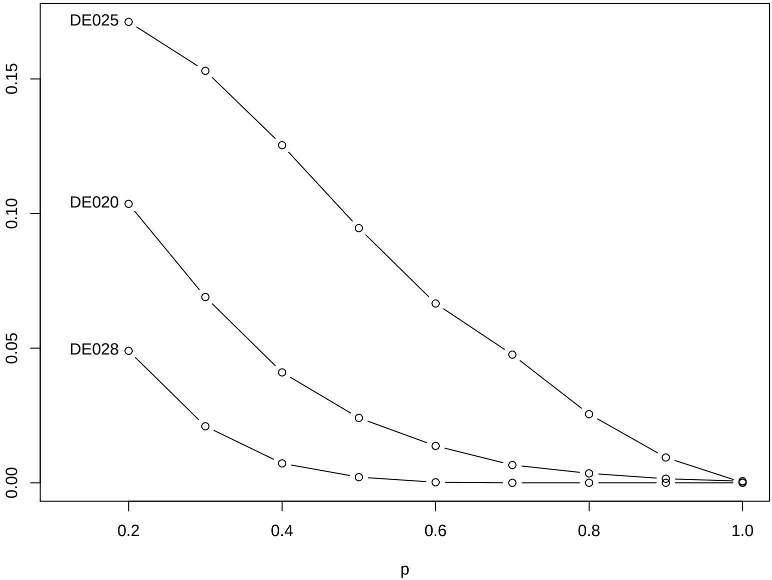
Balance sheet data (in million Euros) from banks in the EBA 2011 stress test:

| Bank code | Bank | $a^{(e)} + a$ | $a$ | $w$ |
|---|---|---|---|---|
| DE017 | DEUTSCHE BANK AG | 1,905,630 | 47,102 | 30,361 |
| DE018 | COMMERZBANK AG | 771,201 | 49,871 | 26,728 |
| DE019 | LANDESBANK BADEN-WURTTEMBERG | 374,413 | 91,201 | 9,838 |
| DE020 | DZ BANK AG | 323,578 | 100,099 | 7,299 |
| DE021 | BAYERISCHE LANDESBANK | 316,354 | 66,535 | 11,501 |
| DE022 | NORDDEUTSCHE LANDESBANK -GZ- | 228,586 | 54,921 | 3,974 |
| DE023 | HYPO REAL ESTATE HOLDING AG | 328,119 | 7,956 | 5,539 |
| DE024 | WESTLB AG, DUSSELDORF | 191,523 | 24,007 | 4,218 |
| DE025 | HSH NORDBANK AG, HAMBURG | 150,930 | 4,645 | 4,434 |
| DE027 | LANDESBANK BERLIN AG | 133,861 | 27,707 | 5,162 |
| DE028 | DEKABANK DEUTSCHE GIROZENTRALE | 130,304 | 30,937 | 3,359 |

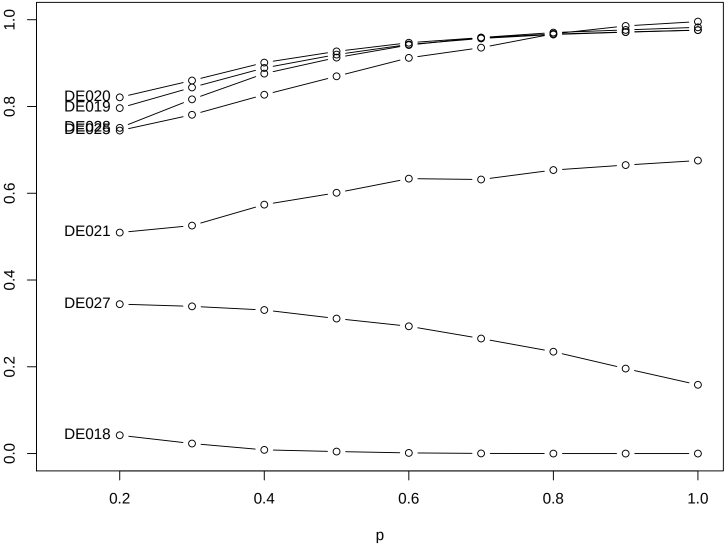
# Stress testing

- Deterministic shock to external assets of all 11 banks in the network by reducing external assets by 3%.
- Shock causes fundamental default of 4 banks: DE017, DE022, DE023, DE024.
- We apply the clearing approach by Eisenberg & Noe (2001) and [Rogers & V. (2013)] to determine which banks suffer contagious defaults.
- Gibbs sampler allows us to derive posteriori default probabilities for remaining 7 banks.

# Default probabilities of banks as a function of $p$



# Default probabilities for clearing with default costs




(a) Clearing with $\alpha = 1, \beta = 0.7$

# Mean out-degree of banks, i.e., $\mathbb{E}[\sum_j \mathcal{A}_{ij} \mid a, l]$, for different $p^{\mathrm{ER}}$ in the Erdős-Rényi network

| | l | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|
| DE020 | 99936 | 3.50 | 4.40 | 5.40 | 6.20 | 6.90 | 7.60 | 8.30 | 9.00 | 10.00 |
| DE019 | 91314 | 3.30 | 4.20 | 5.10 | 6.00 | 6.70 | 7.50 | 8.20 | 8.90 | 10.00 |
| DE021 | 66494 | 2.90 | 3.70 | 4.70 | 5.50 | 6.40 | 7.20 | 8.00 | 8.80 | 10.00 |
| DE022 | 54907 | 2.70 | 3.50 | 4.40 | 5.30 | 6.10 | 7.00 | 7.80 | 8.80 | 10.00 |
| DE018 | 49864 | 2.60 | 3.40 | 4.30 | 5.10 | 6.00 | 6.90 | 7.80 | 8.70 | 10.00 |
| DE017 | 46989 | 2.50 | 3.30 | 4.20 | 5.10 | 5.90 | 6.80 | 7.70 | 8.70 | 10.00 |
| DE028 | 30963 | 2.20 | 2.80 | 3.60 | 4.50 | 5.40 | 6.30 | 7.30 | 8.40 | 10.00 |
| DE027 | 27679 | 2.10 | 2.70 | 3.50 | 4.30 | 5.20 | 6.10 | 7.10 | 8.30 | 10.00 |
| DE024 | 23971 | 1.90 | 2.60 | 3.30 | 4.10 | 5.00 | 5.90 | 7.00 | 8.20 | 10.00 |
| DE023 | 8023 | 1.40 | 1.80 | 2.30 | 2.80 | 3.50 | 4.30 | 5.40 | 6.90 | 10.00 |
| DE025 | 4841 | 1.20 | 1.50 | 1.90 | 2.40 | 2.90 | 3.60 | 4.60 | 6.10 | 10.00 |

# Hierarchical Models

- The basic model can be used as a building block in more complicated models, e.g. in hierarchical models:

$$\begin{aligned} \theta &\sim \pi(\theta), \\ (p_{ij}, \lambda_{ij})_{i,j\in\mathcal{N}} &= f(\theta), \end{aligned} \tag{3}$$

where $\pi$ is an a-priori distribution on $\theta$ and $f$ is a given function.

- Sampling of $\theta, L|l, a, L^*$ combined by iterating between sampling
- $L|\theta, l, a, L^*$ (using the Gibbs sampler)
- and $\theta|L$ (using more standard MCMC techniques, eg. Gibbs, Metropolis Hasting).

# Example: Conjugate distribution model

$p$ and $\lambda$ consist of identical but random values;
$\theta = (\tilde{p}, \tilde{\lambda})$

$$\tilde{p} \sim \text{Beta}(a, b), \quad \tilde{\lambda} \sim \text{Gamma}(c, d)$$
$$p_{ij} = \tilde{p}\mathbb{I}(i \neq j), \quad \lambda_{ij} = \tilde{\lambda}, \quad i, j \in \mathcal{N}.$$

for some parameters $a, b, c, d$.

- prior on $\tilde{p}$, $\tilde{\lambda}$ is flexible,
- direct sampling of $\theta | L$ possible (conjugate distributions).
- Extensions: Partition $p, \lambda$, use independent models for partitions.

# Fitness model - Power law in Degree Distr & Weights

- Empirical literature (Boss et al., 2004, e.g.) suggests that power laws are reasonable models for degree distributions AND for liabilities, i.e. their density would be of the form

$$p(x) = cx^{\alpha}$$

  Empirical studies often find $\alpha$ between $-2$ and $-3$.

- Servedio et al. (2004): fitness-based model for degree distributions.
- We will couple such a fitness model for the degrees with a model for liabilites that allows a power law. Key ideas
  - Use same fitness for both liabilities and degrees.
  - Gamma mixture of exponential distributions lead to a Pareto type II distribution (also called Lomax distribution)

# Fitness model - Power law in Degree Distr & Weights

$$
\begin{aligned}
X_i &\sim \mathrm{Exp}(1), i \in \mathcal{N}, \\
p_{ij} &= f(X_i + X_j)\mathbb{I}(i \neq j), \quad i,j \in \mathcal{N}, \\
\lambda_{ij} &= G^{-1}_{\zeta,\eta}(\exp(-X_i)) + G^{-1}_{\zeta,\eta}(\exp(-X_j)), \quad i,j \in \mathcal{N} \\
(\zeta,\eta) &\sim \pi(\zeta,\eta),
\end{aligned}
$$

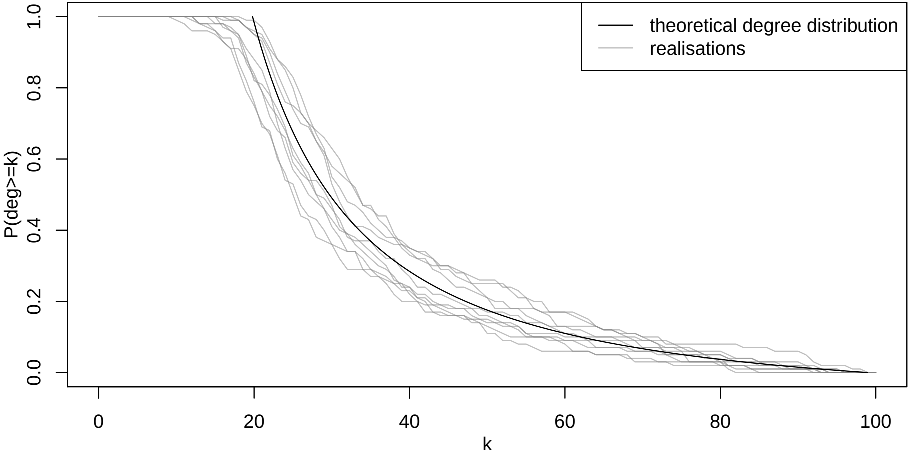
where $G^{-1}_{\zeta,\eta}$ is the quantile function of a Gamma distr. with parameter $\zeta > 0, \eta > 0$. $\pi$ is a prior distribution on $\zeta,\eta)$ and

$$
f(x) := \begin{cases} \beta\left(\frac{\gamma}{\beta}\right)^{1-\exp(-x)}\left(1-\log\left(\frac{\gamma}{\beta}\right)\exp(-x)\right), & \text{if } \alpha = -1, \\ \beta\left(\xi + (1-\xi)e^{-x}\right)^{\frac{1}{\alpha+1}}\left\{1 + \frac{1}{\alpha+1}\frac{1-\xi}{\xi e^x + 1 - \xi}\right\}, & \text{if } \alpha \neq -1, \end{cases}
$$

where $\xi := \left(\frac{\gamma}{\beta}\right)^{\alpha+1}$.

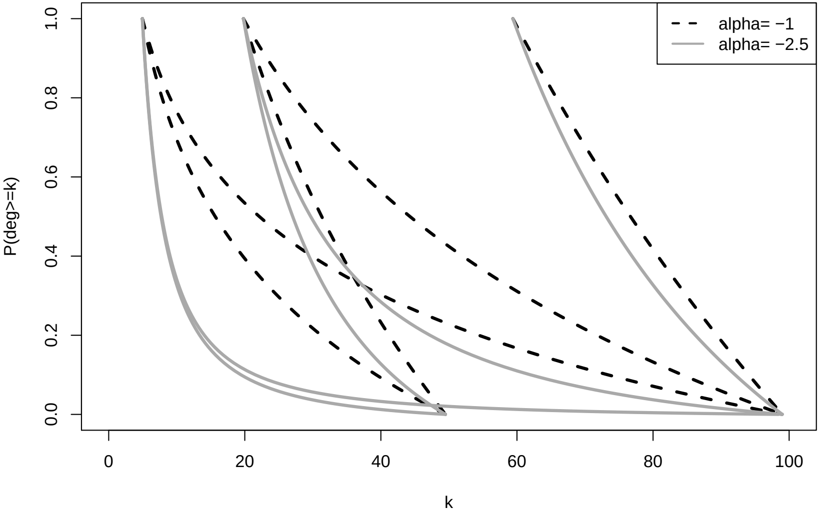
# Realisations of Out Degree Distributions

$n = 100$; $\alpha = -2.5$; $\beta = 0.2$; $\gamma = 1.0$



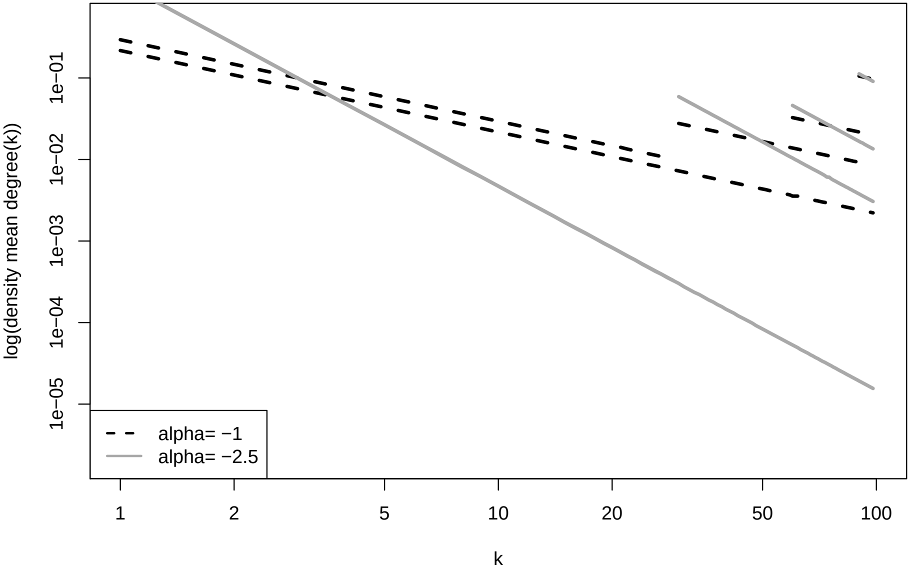

Theoretical dist $\approx$ distr of $\sum_{j=1}^{n} p_{ij}$

# Survival Function - Theoretical Degree Distribution

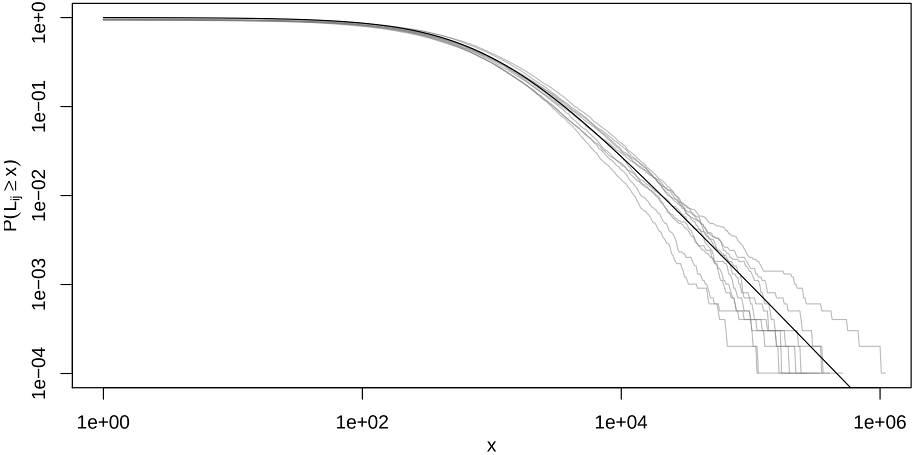



Flexibility in the degree distribution (different choices of $\alpha$, $\beta$, $\gamma$).

# PDF - theoretical out degree for various parameter choices



# Log-log plot of the survival function of $L$




Black: theoretical distribution (type II Pareto)
Gray: Realisations

# Mean out-degree of banks, i.e., $\mathbb{E}[\sum_j \mathcal{A}_{ij} \mid a, l]$

| model | | ER | | Fitness | | | |
|---|---|---|---|---|---|---|---|
| model parameters | | $p^{\mathrm{ER}}$ | | $\alpha, \beta, \gamma$ | | | |
| name | l | 0.5 | 0.9 | -2.5,0.2,1 | -2.5,0.2,0.6 | -2.5,0.5,1 | -1,0.5,1 |
| DE020 | 99936 | 6.20 | 9.00 | 8.80 | 6.10 | 9.40 | 9.60 |
| DE019 | 91314 | 6.00 | 8.90 | 8.50 | 5.80 | 9.20 | 9.40 |
| DE021 | 66494 | 5.50 | 8.80 | 7.50 | 5.30 | 8.70 | 9.00 |
| DE022 | 54907 | 5.30 | 8.80 | 6.90 | 4.90 | 8.40 | 8.70 |
| DE018 | 49864 | 5.10 | 8.70 | 6.70 | 4.80 | 8.30 | 8.60 |
| DE017 | 46989 | 5.10 | 8.70 | 6.60 | 4.70 | 8.20 | 8.60 |
| DE028 | 30963 | 4.50 | 8.40 | 5.70 | 4.20 | 7.60 | 8.00 |
| DE027 | 27679 | 4.30 | 8.30 | 5.50 | 4.00 | 7.40 | 7.80 |
| DE024 | 23971 | 4.10 | 8.20 | 5.30 | 3.90 | 7.30 | 7.70 |
| DE023 | 8023 | 2.80 | 6.90 | 4.00 | 3.10 | 6.30 | 6.60 |
| DE025 | 4841 | 2.40 | 6.10 | 3.60 | 2.70 | 5.90 | 6.30 |
| posteriori | | 4.66 | 8.25 | 6.30 | 4.50 | 7.90 | 8.20 |
| a-priori | | 5.00 | 9.00 | 3.60 | 3.10 | 6.80 | 7.20 |

# Summary

- Bayesian setup can be used to fill in "missing" information in a principled way. Large flexibility.
- Construction of Gibbs sampler for sampling from distribution of liabilities matrix conditional on its row and column sums.
  R package (systemicrisk) available from CRAN.
  Some theoretical results + simulation studies show that sampler works.
- Can be used for stress tests using empirical data.
- Can be extended to incorporate additional information such as expert views etc. on the network structure:
  - Hierarchical model for $p$, $\Lambda$ ($\rightarrow$ power laws).
  - Observation of some components of the matrix.

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# Identifiability

The matrices $p$ and $\lambda$ cannot be fully identified merely by observing the row and column sums.

## Lemma

*Let $l, a \in (0,\infty)^n$ with $A = \sum_{i=1}^n l_i = \sum_{i=1}^n a_i$ and $a_i + l_i < A$ for all $i$. Let $p \in [0,1]^{n\times n}$ with $p_{ij} > 0\ \forall i \neq j$, $diag(p) = 0$. Then*

$$\exists(\lambda_{ij}) : \forall j : \sum_{i=1}^n \mathbb{E}(L_{ij}) = a_j \text{ and } \sum_{j=1}^n \mathbb{E}(L_{ij}) = l_j.$$

Hence: need to make assumptions about $p$.