Taking the One-Year Change from Another Angle

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LUH-Kolloquium "Versicherungs- und Finanzmathematik", Hannover, Nov. 28, 2014

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Collaboration

- This work was done in collaboration with A. Ferriero and D. Krief*

*) M. Dacorogna, A. Ferriero and D. Krief, *Taking the one year change from another angle,* submitted for publication 2014

Agenda

The Problem (1/2)

- Insurance companies keep reserves to guaranty payment of the losses generated by the contracts they signed
- Economic valuation of reserves are made of a *best estimate* (based on the market value of the replicating portfolio) of the reserves plus a *risk margin*
- The uncertainty of insurance liability cash flows forces to introduce a risk margin equal to the expected cost of having to hold solvency capital for non-hedgeable risk (cash flows generated by risk that cannot be replicated by financial instruments) during the life-time of the contract
- The knowledge of the risk-adjusted capital over time is necessary to compute the risk margin as the cost of this capital
- Moreover, insurance companies under Solvency II or SST (Swiss Solvency Test) must compute their Solvency Capital Requirement as the capital necessary to cover fluctuations over one *calendar year*

The Problem (2/2)

- The calendar year capital for SST and Solvency II is based on the *yearly fluctuations of the reserves* and not on the ultimate losses
- Regulators are concerned about the company *surviving one year* and being able to run-off or sell its liabilities
- That is why they want the company to hold also a risk margin that would be required for an economic valuation of the reserves and thus by the buyer
- The calendar year capital is based also on the *one year uncertainty* of the ultimate reserves of insurance liabilities

Formal Definitions of the One Year Change

- Consider a n-step loss process leading to an ultimate loss

- We denote by $U(n)$ the ultimate loss and by F_i the information available at time i, for $i \in \{0, \dots, n\}$. The *one-year change in reserves* can be evaluated using the formula

$$D(i) = E(U(n)|F_i) - E(U(n)|F_{i-1}) \quad \text{for } i \in \{1, \dots, n\}$$

- After some simplifications, the solvency capital requirement associated with year i can be computed as

$$C_i = E(\rho(D(i)|F_{i-1})|F_0),$$

where ρ is the chosen *risk measure*. Generally, $\rho = \mathrm{TVaR}_{99\%}$ (SST) or $\rho = \mathrm{VaR}_{99.5\%}$ (Solvency II). For this study, we choose the first one.

Formal Definitions of the Risk Margin

- The risk margin R_n is then defined as the cost of the required capital

$$R_n = \eta \sum_{j=1}^{n} C_i,$$

where η designates the cost of capital, which corresponds to the expected return for investing in a risky asset. Generally, $\eta = 6\%$ above the risk free rate, but we use here $\eta = 10\%$

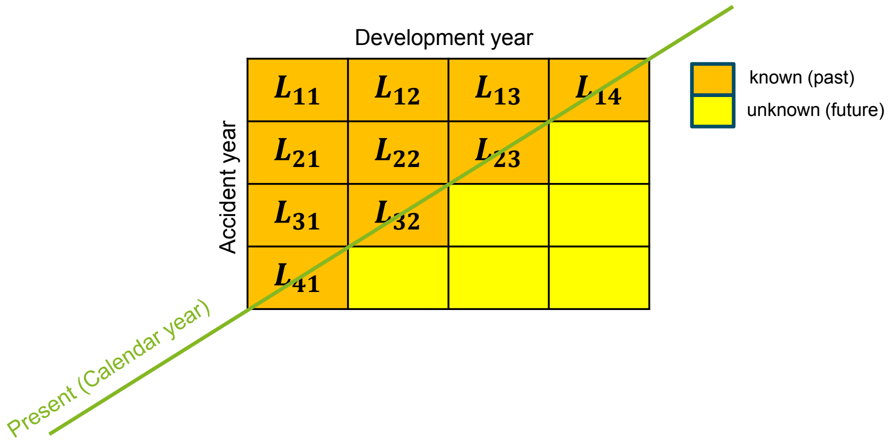
- For sake of simplicity, we omit the discount of the risk margin

- In particular, the risk of the first year, i.e. $C_1 = \rho(D(1)|F_0)$ is the solvency capital

- It is the quantity that is required both by Solvency II and SST for computing the SCR and the RBC respectively

The one year change

- In practice, for P&C insurance, these risk measurements are not done for one “accident” year, but on a triangle.



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Merz-Wüthrich Approach to the One Year Change

- The Merz-Wüthrich method[1] gives the one-year risk of a P&C insurance contract in form of the conditional standard deviation of the first one-year change

$$\mathrm{std}(D(1)|F_0)$$

- It uses the Mack method[2] to calculate the ultimate uncertainty. It relies on the same assumptions for the underlying process
- Mack makes the following assumptions:
  - The rows of the triangle are independent
  - There are development factors $\exists\, f_1, \ldots, f_{n-1} > 0$, such that
    $E\left(L_{j,i+1} \middle| F_{j,i}\right) = f_i\, L_{j,i}$
  - And consequently $\exists\, \sigma_1, \ldots, \sigma_{n-1} > 0$, such that $Var\left(L_{j,i+1} \middle| F_{j,i}\right) = \sigma_i^2\, L_{j,i}$

1) M. Merz, M. V. Wüthrich, Modelling the claims development result for solvency purposes, Casualty Actuarial Society (2008)
2) T. Mack, Distribution-free calculation of the standard error of the chain ladder reserve estimates, Astin Bulletin, vol. **23**, 213-225 (1993)

Merz-Wüthrich Method a Simple and Good Idea

- The formulae are rather complicated, but the idea is simple:
  - use the Mack method to create the following triangle out of the initial one

L_{11}	L_{12}	L_{13}	L_{14}
L_{21}	L_{22}	L_{23}	
L_{31}	L_{32}		
L_{41}			

$\rightarrow$

L_{11}	L_{12}	L_{13}	L_{14}
L_{21}	L_{22}	L_{23}	L_{24}
L_{31}	L_{32}	L_{33}	
L_{41}	L_{42}		

  - Then look at the variations between the two diagonals
  - Thus, the method carries out the same assumptions as the Mack method

The Capital Over Time (COT) Method (1/2)

- The *Capital Over Time* (COT) method* is the method currently used at SCOR to estimate the risk margin. It calculates an approximation of the solvency capital requirement by *scaling the ultimate risk* with the formula

$$K_i = \delta_i \, \rho(U(n)|F_0),$$

where $\rho = \mathrm{xTVaR}_{99\%} \equiv \mathrm{E}[L] - \mathrm{TVaR}_{99\%}[L]$.

- The vector $\delta = (\delta_1, \dots, \delta_{n-1})$ is called *COT-pattern* and is calculated using the following formula:

$$\delta_i = \gamma_i^b (1 - p_b) + p_b \sum_{j=i}^{n} \gamma_j,$$

where b and p_b are parameters of the model and γ designates incremental calendar pattern

*) A. Ferriero, *Solvency capital estimation, reserving cycles and ultimate risk*, preprint 2014

The Capital Over Time (COT) Method (2/2)

- The idea behind the method is that the development process is a *geometric Lévy-like process* composed of two processes: a good “continuous” part (attritional losses) and a bad “jump” part (large losses)
- Parameter p_b designates the contribution of the bad process
- Parameter b is a dependence parameter. Its value depends on the length of the claim developments in the particular line of business

$$\begin{array}{lll} b = 0.60 & \text{for short tail,} & (\text{m.t.p.} < 2) \\ b = 0.65 & \text{for medium tail,} & (2 \le \text{m.t.p.} \le 4) \\ b = 0.75 & \text{for long tail,} & (\text{m.t.p.} > 4). \end{array}$$

Changing the Perspective

- In this study, we have decided to change the perspective: instead of *starting from the ultimate* to deduce the calendar year change, we formulate a time dependent process that *leads to the ultimate*
- We look at “simple” theoretical development processes that reflect some logical features of a loss process
- We design simple time dependent processes to be able to obtain *explicit formulae* for the calendar year changes
- Then, we simulate these processes and *compare the results* obtained with the Merz-Wüthrich and COT methods in order to see how well they are able to cope with such information and give reasonable answers
- In a next step, we would like to find ways to parameterize these processes to model the *real loss processes* that lead to the ultimate loss

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Linear Model (1/3)

- The first model studied is an extension of the process presented in Busse et al. 2014* to a multi-step framework
- We throw n dices in n steps. At each step, we choose randomly a number of throws according to a uniform distribution over all the remaining possibilities, i.e.,

$$\mathbb{P}\left[N_i = n_i \mid (N_j = n_j, 1 \le j \le i-1)\right] = \frac{1}{1 + n - \sum_{j=1}^{i-1} n_j}$$

- The philosophy behind this model is that what has already been paid need not to be paid anymore and what has not been paid sooner need to be paid later. The changes are *linear*

*) M. Busse, M. Dacorogna, M. Kratz; The impact of systemic risk on the diversification benefits of a risk portfolio, Risks, vol. 2, p. 260-275 (2014)

Linear Model (2/3)

- We start from time t_0 to develop the process to the ultimate t_n*
- Our loss is then 1 for each “6” obtained. The loss at step i is therefore distributed as $(S_{N_i}^{(i)} | N_i) \sim B(N_i, p)$, with p=1/6

$\mathbf{0}$	$\boldsymbol{S}^{(1)}$	$\boldsymbol{S}^{(2)}$	$\boldsymbol{S}^{(3)}$	$\boldsymbol{S}^{(4)}$	$\boldsymbol{S}^{(5)}$	$\boldsymbol{S}^{(6)}$	$\boldsymbol{S}^{(7)}$	$\boldsymbol{S}^{(8)} = \boldsymbol{U}(\boldsymbol{n})$

$+S_{N_1}^{(1)}$ $+S_{N_2}^{(2)}$ $+S_{N_3}^{(3)}$ $+S_{N_4}^{(4)}$ $+S_{N_5}^{(5)}$ $+S_{N_6}^{(6)}$ $+S_{N_7}^{(7)}$ $+S_{N_8}^{(8)}$

- The ultimate distribution is $U(n) \sim B(n, p)$

*) M. Dacorogna, A. Ferriero, D. Krief; *Taking the one year change from another angle,* submitted for publication to the ASTIN Bulletin, June 2014

Linear Model (3/3)

- The advantage of such a process, is that one can calculate explicitly many quantities such as:
  - $D(i) = S_{N_i}^{(i)} - N_i p$
  - $E(N_i) = \frac{n}{2^i}$ for $i \in \{1, \dots, n-1\}$ and $E(N_n) = \frac{n}{2^{n-1}}$
  - Incremental development pattern is $\gamma_i = 2^{-i}$ for $i \neq n$ and $\gamma_n = 2^{1-n}$
  - (Un)conditional expectation and covariance of the $D(i)$. Note in particular that the $D(i)$'s can be shown in general to be uncorrelated
  - An n-step process terminates on average in

$$E(T_n) \approx 1 + \sum_{j=1}^{n} \frac{1}{j} \approx \log(n) + 1.57.$$

Multiplicative Model (1/3)

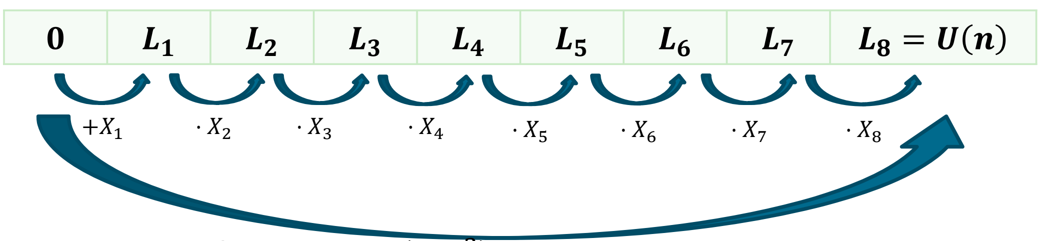
- The second model is developed because the ultimate losses of some lines of business are generally considered as being log-normally distributed. Assume that we know the ultimate distribution

$$U(n) \sim LogN(\mu, \sigma^2)$$

- The loss model is defined as a product of intermediate losses

$$L_i = X_1 \cdot \ldots \cdot X_i \sim \text{Log-}\mathcal{N}\left(\sum_{j=1}^{i} \mu_j \ , \ \sum_{j=1}^{i} \sigma_j^2\right)$$

where $X_i \sim LogN(\mu_i, \sigma_i^2)$, independently



Obtain $LogN(\mu, \sigma^2)$ ultimate distribution.

Multiplicative Model (2/3)

- In particular, we have:

$$U(N) = L_N \sim \text{Log-}\mathcal{N}\left(\sum_{j=1}^{N} \mu_j \, , \, \sum_{j=1}^{N} \sigma_j^2\right)$$

- This gives the two equations:

$$\begin{cases} \sum_{j=1}^{N} \mu_j & = \mu \\ \sum_{j=1}^{N} \sigma_j^2 & = \sigma^2 \end{cases}$$

- and $2n$ parameters!!!
- How to fit the $2n$ parameters of such a model?

Multiplicative Model (3/3)

- We use the cumulative development pattern $\gamma = (\gamma_1, \ldots, \gamma_n)$, which is usually available and fit the parameters of the model to match the chosen pattern

$$\frac{E(L_i)}{E\big(U(n)\big)} = e^{-\mu - \frac{\sigma^2}{2} + \sum_{j=1}^{i} \mu_j + \sigma_j^2/2} = \gamma_i$$

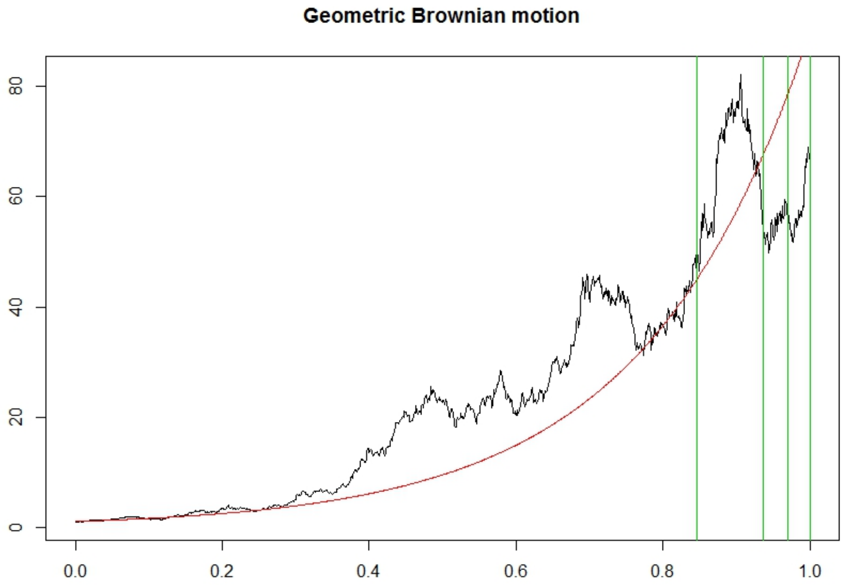
- This is not sufficient. We therefore add another condition that the ratio between σ_i^2 and μ_i is constant at every steps and is equal to $2k$:

$$\exists k > 0, \text{such that } \sigma_i^2 = 2k\mu_i$$

- Then by choosing $\begin{cases} \mu_1 = \mu + \frac{\log(\gamma_1)}{1+k} \\ \mu_i = \frac{\log(\gamma_i) - \log(\gamma_{i-1})}{1+k} \, , \qquad 2, \ldots, N \end{cases}$

- we obtain a model with the desired ultimate distribution and the desired pattern, and equivalent to a Geometrical Brownian Motion

Illustration of the Multiplicative Model



Geometric Brownian motion path with drift $\mu = 4$ and volatility $\sigma^2 = 1$. The red curve shows the expectation of the process. The green lines represent the stopping times corresponding to the pattern $\gamma = (0.5, 0.75, 0.875, 1)$.

Advantages of the Multiplicative Model (1/2)

- The multiplicative model, like the linear model, has many interesting properties that can be computed explicitly
- The main one, is that the conditional and unconditional distributions are log-normally distributed. In particular, the moments are the same, which makes computing them easy
- The one-year changes are in this case (unconditionally) the difference of two log-normal random variables

$$D(i) = X_1 \cdot \ldots \cdot X_{i-1} \cdot \left(X_i - E(X_i)\right) \cdot E(X_{i+1}) \cdot \ldots \cdot E(X_n)$$

- But conditionally on the last step (F_{n-1}), they follow a centered log-normal distribution.

Advantages of the Multiplicative Model (2/2)

- We can compute explicitly the capital over time for TVaR:

$$C_i(\mathrm{TVaR}_\alpha) = \mathbb{E}\left[\mathrm{TVaR}_\alpha(D(i)|\mathcal{F}_{i-1})\right] = \left[\frac{\Phi\left(\sigma_i - \Phi^{-1}(\alpha)\right)}{1-\alpha} - 1\right] \cdot e^{\mu+\frac{\sigma^2}{2}}$$

- and hence the risk margin for TVaR:

$$R_N(\mathrm{TVaR}_\alpha) = \eta \sum_{i=1}^{N} C_i(\mathrm{TVaR}_\alpha) = \eta e^{\mu+\frac{\sigma^2}{2}} \left[\sum_{i=1}^{N} \frac{\Phi\left(\sigma_i - \Phi^{-1}(\alpha)\right)}{1-\alpha} - N\right]$$

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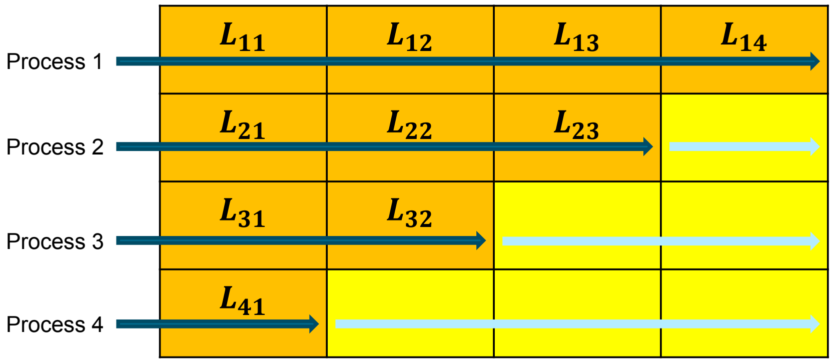
Method to Compute the One Year Capital (1/3)

- For both processes, our methodology is similar. We consider triangles for which the losses of each underwriting year are driven independently by the process. Note that the different representations of the process are at different stages of development



Method to Compute the One Year Capital (2/3)

- To clarify the notation between single line values and triangle values, we introduce the following notations for one-year change and the yearly capital requirements of a triangle

$$\Delta(i) = \sum_{j=i+1}^{n} D_j(n+i+1-j)\,,$$
$$K_i = E(\rho(\Delta(i)|F_{i-1})|F_0), \qquad i \in \{1, \dots, n-1\}$$

where $D_j(k)$ designates the k^{th} one-year change of line j and F_i the information available above the i^{th} diagonal after present

- The corresponding risk margin is

$$R_n = \eta \sum_{j=1}^{n-1} K_j\,.$$

Method to Compute the One Year Capital (3/3)

- For both models, we simulate a "large" number of triangles, on which we calculate the capital requirement of the first year using the Merz-Wüthrich method, the COT method and a benchmark method that is either a theoretical value or a numerical approximation of it
- We calculate each time the mean value of the capital requirement over all triangles, the standard deviation around that mean
- We also calculate the following measures of deviation from the benchmark value:
  - Mean absolute deviation (MAD): $E(|\text{estimate} - \text{true}|)$
  - Mean relative absolute deviation (MRAD): $E\left(\frac{|\text{estimate} - \text{true}|}{\text{true}}\right)$
  - Mean relative deviation (MRD): $E\left(\frac{\text{estimate} - \text{true}}{\text{true}}\right)$ (when useful)

The One Year Capital for the Linear Model (1/3)

- For the linear model, the benchmark is calculated using a normal approximation
- Since $D(i) = S_{N_i}^{(i)} - N_i\, p = S_{N_i}^{(i)} - E\left(S_{N_i}^{(i)}\right)$,
- the first one-year change $\Delta(1)$ of a triangle has the F_0-conditional distribution of a "centered mixture of binomial distribution", where the mixture distribution is a sum of discrete uniforms distribution.
- This is the distribution that we approximate by a normal distribution with same expectation (zero) and variance (known). We then use the fact that, for a normal random variable $X \sim N(\mu, \sigma^2)$,

$$\text{TVaR}_\alpha(X) = \mu + \sigma \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}.$$

The One Year Capital for the Linear Model (2/3)

- For the Merz-Wüthrich method, we use the same normal approximation to convert conditional standard deviation into conditional TVaR
- Additional problem: The Merz-Wüthrich method, being multiplicative, it fails if there is a 0 in the triangle, which happens often with the linear model. We therefore use the trick that consists in taking a truncated version of a linear process generated triangle with n much larger.
- The truncation is made after

$$\log(n) + 6.57 \approx E(T_n) + 5$$

- steps to insure that the process is almost finished, thus making the remaining risk negligible.

The One Year Capital for the Linear Model (3/3)

- For the COT method, we do not need to convert a standard deviation into a TVaR. However, we need to calculate the ultimate risk. This is done by simulating from the (know) binomial distribution and taking the empirical TVAR.
- Recall the COT formula

$$K_i = \delta_i \,\mathrm{xTVaR}_{99\%}(U(n)|F_0),$$

$$\delta_i = \gamma_i^b(1 - p_b) + p_b \sum_{j=i}^{n} \gamma_j.$$

- The parameters of this formula were designed for real data with dependences and jumps. In the case of our artificial process, it is not obvious that they are the same. We therefore apply the COT method in two ways:
  - With jumps: p_b from the formula and $b = 0.75$ (for long tail)
  - Without jumps: $p_b = 0$ and $b = 0.5$

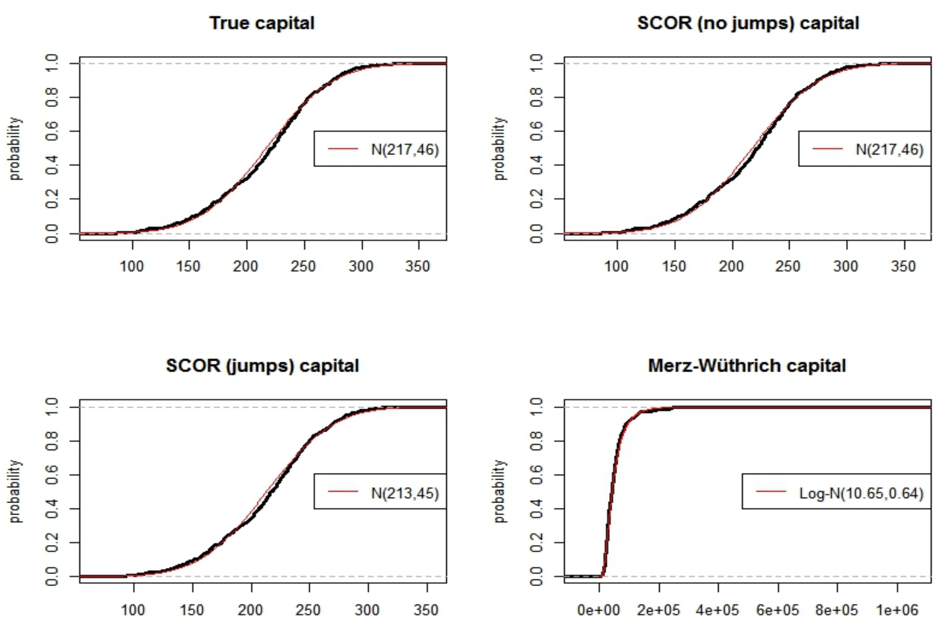
Results for the One Year Capital of the Linear Model

- We make the comparison on 500 triangles and obtain the following results

Method	Mean	Std. dev.	MAD	MRAD	Rob. MAD	Rob. MRAD
Benchmark	216.85	45.75	0	0%	0	0%
SCOR, no jumps	217.34	45.78	0.79	0.38%	0.74	0.35%
SCOR, jumps	213.38	44.94	3.48	1.68%	3.44	1.63%
Merz-Wüthrich	53’621	59’075	53’404	25’424%	45’209	21’795%

- The COT method gives excellent results. In particular, without jump, which is to be expected. But with jumps also, the results are very satisfactory. The Merz-Wüthrich method, however, completely fails even in estimating the order of magnitude of the capital

CDF* of the Linear Model; a Comparison



*) Empirical distributions of the different first year capital estimations with fitted distributions (method of moments)

Discussion of the Results of the Linear Model

- From looking at triangles, one can get an intuition for the reason of the failure. The Merz-Wüthrich method is multiplicative. Therefore, it considers that any value is as likely to be multiplied by a large number. The linear model however is an additive model, which makes small values more likely to be “multiplied” by a large factor

- In addition, the Mack hypotheses assume that $Var\left(L_{j,i+1} \middle| F_{j,i}\right) = \sigma_i^2 \, L_{j,i}$. Hence, the larger the previous loss, the more volatility there is in the remaining process. For the linear model however, due to the “fixed number of dices” property, it is the opposite. Indeed, a large previous loss means that a large number of dices has most likely been already thrown, which implies less remaining risk. This explains the negative correlation

Extreme Example of Simulated Triangles (1/2)

- An extreme example of the first phenomenon is the following:

Triangle													
35	67	130	143	162	178	179	179	185	186	186	186	186	186
40	80	155	163	164	164	164	164	164	164	164	164	164	
92	131	153	153	156	160	161	162	162	162	162	162		
81	140	149	156	163	166	172	175	175	175	175			
49	71	120	163	170	170	173	173	173	173				
154	172	173	174	174	174	174	174	174					
65	129	155	157	157	157	157	157						
51	64	166	166	166	166	166							
153	153	156	159	162	162								
153	158	159	159	159									
1	97	152	158										
84	105	113											
93	106												
130													

- The true first year capital is 22.01 while Merz-Wüthrich would give 1’233.67

Extreme Example of Simulated Triangles (2/2)

Triangle													
35	67	130	143	162	178	179	179	185	186	186	186	186	186
40	80	155	163	164	164	164	164	164	164	164	164	164	
92	131	153	153	156	160	161	162	162	162	162	162		
81	140	149	156	163	166	172	175	175	175	175			
49	71	120	163	170	170	173	173	173	173				
154	172	173	174	174	174	174	174	174					
65	129	155	157	157	157	157	157						
51	64	166	166	166	166	166							
153	153	156	159	162	162								
153	158	159	159	159									
50	97	152	158										
84	105	113											
93	106												
130													

If we say that half of the dices of the critical case's second step were thrown on the first step, the true first year capital is still 22.01. The Merz-Wüthrich one however is now 209.09, which divides by 6 the one of the previous case

The One Year Capital for the Multiplicative Model

- Here, it is not possible to switch easily from standard deviation to TVaR because in the triangle we have to sum up various log normal at different stages of development. We therefore use directly standard deviation as a risk measure for comparison

- Because the rows are independent, we can directly use the exact conditional standard deviation as benchmark

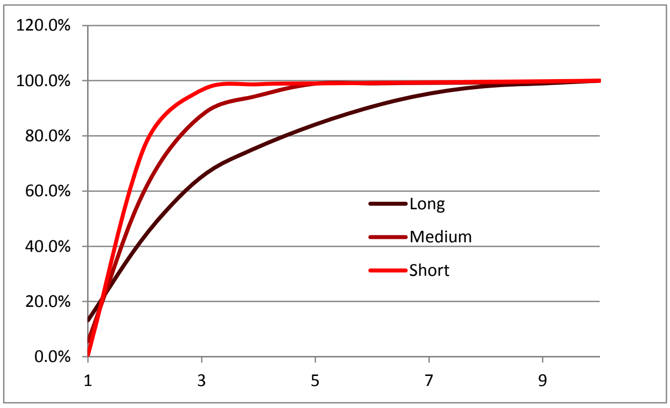
- Since this model is multiplicative, we avoid all the cases that would make the Merz-Wüthrich method fail (such as zeros or even negative values)

- For the COT method, we again use the versions with and without jumps in the same way as for the linear model. The difference is that this time $\rho = \mathrm{std}$, instead of $\mathrm{xTVaR}_{99\%}$. This gives slightly different relative results but in a reasonable proportion

Patterns for the Multiplicative Model

- Since the log-normal process depends on a pattern, we make the capital requirement comparison with 3 different patterns taken from the SCOR portfolio, one short, one medium and one long-tailed. We fix the parameters of the ultimate distribution to $\mu = 10$ and $\sigma^2 = 1$.
- The three development patterns are:

Short	Medium	Long
0.7%	5.6%	13.2%
76.5%	60.5%	43.7%
96.6%	87.5%	65.2%
98.7%	94.6%	76%
99%	98.9%	84.1%
99.2%	99%	90.6%
99.4%	99.2%	95.3%
99.6%	99.3%	98%
99.8%	99.6%	99%
100%	100%	100%



Results for the Multiplicative Model

- The results presented here are for the three different patterns:

Pattern	Method	Mean	Std. dev	MAD	MRAD	MRD
Short	Benchmark	29'263	21'968	--	--	--
	COT, no jumps	26'751	19'839	2'541	8.19%	-7.93%
	COT, jumps	28'295	20'977	1'073	3.48%	-2.56%
	Merz-Wüthrich	22'818	15'773	12'668	43.2%	-5.09%
Medium	Benchmark	22'334	16'488	--	--	--
	COT, no jumps	19'942	14'437	2'434	10.1%	-9.59%
	COT, jumps	21'366	15'532	1'098	4.6%	-3.39%
	Merz-Wüthrich	16'347	9'207	8540	34.3%	-12.5%
Long	Benchmark	20'626	15'150	--	--	--
	COT, no jumps	17'852	12'625	2'774	12.5%	-12.5%
	COT, jumps	18'517	13'315	2'109	9.71%	-9.71%
	Merz-Wüthrich	13'692	6'781	7'792	30.4%	-23.6%

Discussion of the Results of the Multiplicative Model

- Again, the 2 COT methods perform relatively well (within less than 10% with jumps and a bit more with no jumps). It almost always *underestimates* the conditional standard deviation but this is due to the fact that the parameters of the COT formula are calibrated for the TVaR as a risk measure instead of standard deviation

- The Merz-Wüthrich method, here, makes on average 30 to 45% of absolute error. For the long-tailed pattern, it is most of the time *underestimation*.

- One should note, in any case, that this time, even though the estimation is bad, the order of magnitude is the correct one. This is due to the fact that the log-normal model fits much better the Mack assumptions. Indeed, 2 out of 3 are verified

Discussion of Mack's Assumptions (1/2)

- Unfortunately, the third assumption is a critical one for our problem:

$$\exists\, \sigma_1, \dots, \sigma_{n-1} > 0, \text{ such that } Var\left(L_{j,i+1} \middle| F_{j,i}\right) = \sigma_i^2\, L_{j,i}.$$

- It says that the future variance of the loss process is proportional to the previous loss whereas in the case of the log-normal process, one can show that

$$Var\left(L_{j,i+1} \middle| F_{j,i}\right) = Var(X_{i+1})\, L_{j,i}^2 = e^{2\mu_{i+1}+\sigma_{i+1}^2}\left(e^{\sigma_{i+1}^2} - 1\right) L_{j,i}^2,$$

- which means that the future variance is proportional to the square of the previous loss. This explains in particular the tendency to underestimate the capital on average, since, in the cases where the losses are big, the Merz-Wüthrich method will underestimate the impact on the future risk.

Discussion of Mack's Assumptions (2/2)

- This is confirmed when simulating triangles and looking at the estimated standard deviation. The cases for which Merz-Wüthrich overestimates the standard deviation are the cases with globally small losses and vice-versa
- The estimated correlation values (not displayed) between the benchmark and the Merz-Wüthrich standard deviation are between 56% and 70%, indicating that the model explains only partially the process

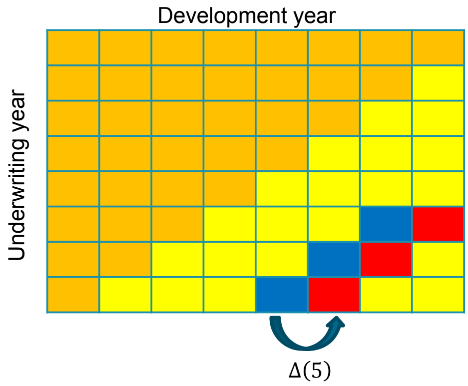
Agenda

Methodology for Computing the Risk Margin (1/2)

- We now discuss risk margin comparison. Merz-Wüthrich does not give the risk margin*. We therefore only compare the results of the COT method to the benchmark risk margin

- Recall the risk margin formula

$$R_n = \eta \sum_{j=1}^{n-1} K_j = \eta \sum_{j=1}^{n-1} E\big(\mathrm{TVaR}_{99\%}(\Delta(j) | F_{j-1}) | F_0\big).$$



*)Since then, Merz and Wüthrich have now a preprint that allows to compute it: *Claims Run-Off Uncertainty: The Full Picture*, November , 2014 available on: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2524352

Methodology for Computing the Risk Margin (2/2)

- For both models, our comparison methodology is again to simulate a large number of triangles (500) from the process in question

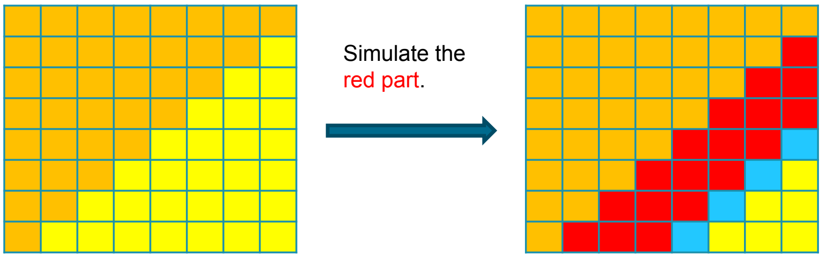
- We then calculate the yearly required capital for each future calendar year with the COT method with or without jump part and with a benchmark (simulation) to compare the results. (We compare graphically the means of the yearly capitals on the 500 triangles)

- We sum the yearly capitals and multiply the result by the cost of capital η=10%

- to obtain the risk margin for each triangle. We compare the results using the same performance statistics as for the one year capital comparison. That is: mean, standard deviation, MAD, MRAD and MRD when useful

Risk Margin for the Linear Model

- We use again the normal approximation to the binomial
- Example for calculation of the benchmark for K_4 on a triangle:

1. 

2. Calculate the standard deviation of $\Delta(4)$ (blue part) using F_3 (orange and red parts).
3. Transform the standard deviation into $\mathrm{TVaR}_{99\%}$ using the normal approximation to obtain an estimate of $\mathrm{TVaR}_{99\%}(\Delta(4)|F_3)$.
4. Repeat $R = 10'000$ times and take the mean to obtain an estimate of

$$K_4 = E(\mathrm{TVaR}_{99\%}(\Delta(4)|F_3)|F_0).$$

Results for the Linear Model

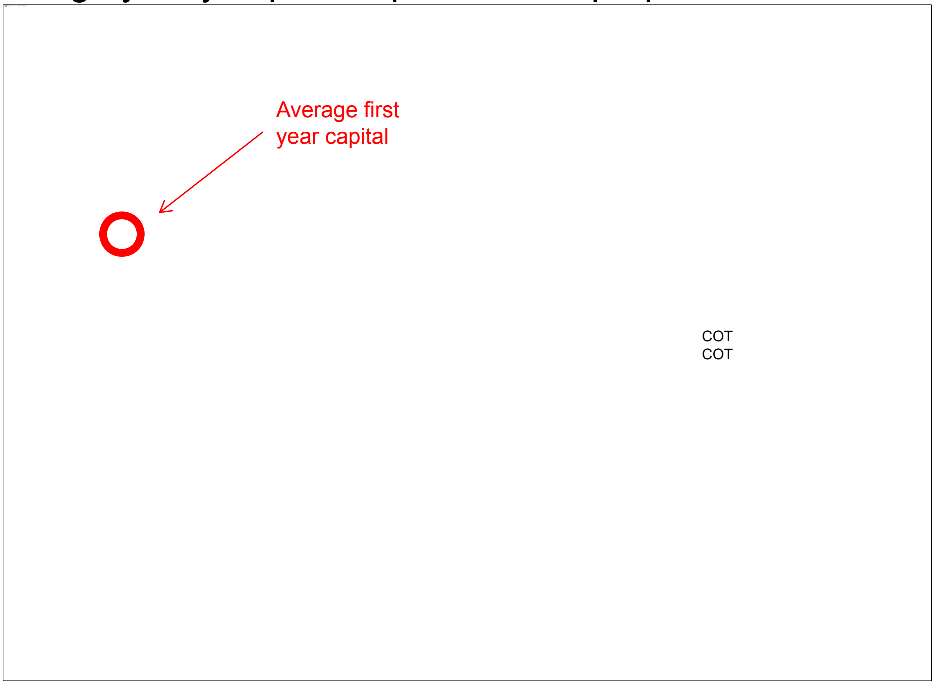
- The COT method behaves exactly as for the one year capital

Method	Mean	Std. dev.	MAD	MRAD
Benchmark	69.55	15.09	--	--
COT, no jumps	74.12	15.97	4.56	6.59%
COT, jumps	49.14	10.58	20.41	29.32%

- The results are rather good for the COT method without jumps. MRD measurement is not necessary since the COT method without (resp. with) jumps always overestimate (resp. underestimate) the risk margin

Capital Over Time in the Linear Model

Average yearly capital requirement in proportion of the ultimate risk

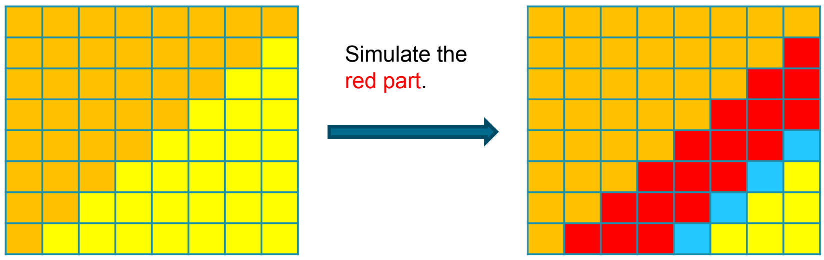


Risk Margin for the Multiplicative Model (1/2)

- For the log-normal model, we again make the comparison with the 3 SCOR patterns: short, medium and long-tail
- The ultimate xTVaR required by the COT method is calculated using simulations, similarly to what is done for the one year capital
- For the benchmark however, we need to use nested simulations, which have the inconvenience of being computationally intensive. This makes, in particular, very difficult to make systematic tests of the stability of a Monte-Carlo estimator
- The reason why nested simulations were not necessary for the linear model is that we had an explicit approximate formula for TVaR, while here we have to sum up log normal distribution, for which there is no explicit formula

Risk Margin for the Multiplicative Model (2/2)

- Example of calculation of the benchmark for K_4 on a triangle:

1. 

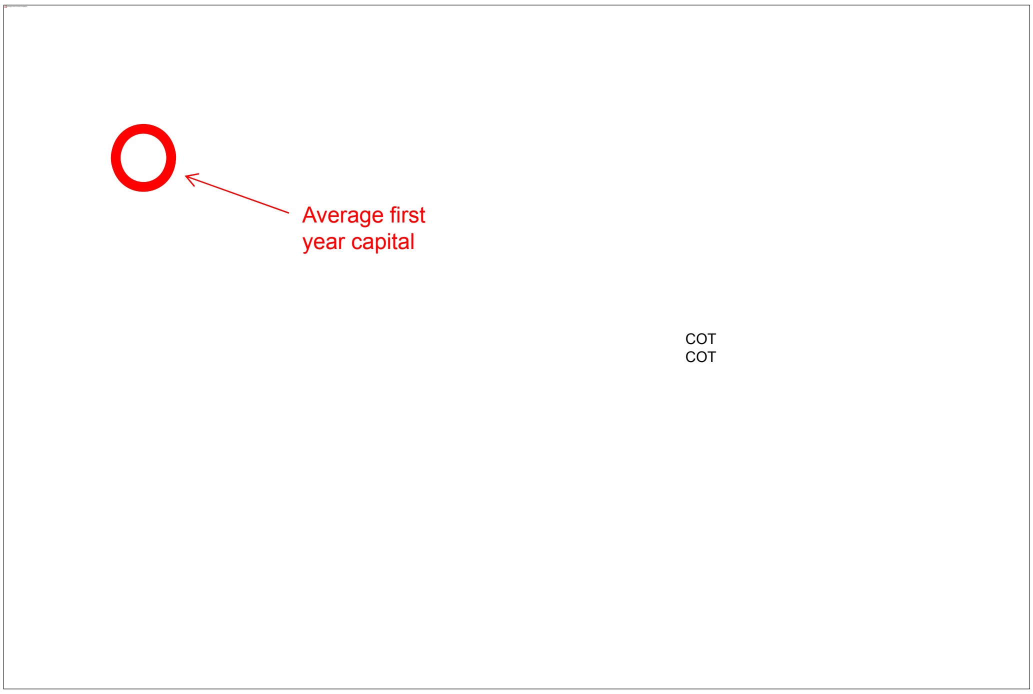
2. Simulate $F_4\backslash\mathrm{F}_3$ (blue part).
3. Calculate explicitly $\Delta(4)$ using F_4 (orange, red and blue parts).
4. Repeat 2 and 3, $R_2 = 1'000$ times and take empirical $\mathrm{TVaR}_{99\%}$ of the sample of calculated $\Delta(4)$ values, thus obtaining an estimate of
$$\mathrm{TVaR}_{99\%}(\Delta(4)|F_3).$$
5. Repeat 1-4, $\mathrm{R}_1 = 50$ times and take the mean to obtain an estimate of
$$K_4 = E(\mathrm{TVaR}_{99\%}(\Delta(4)|F_3)|F_0).$$

Results for the Risk Margin of the Multiplicative Model

Pattern	Method	Mean	Std. dev.	MAD	MRAD	MRD
Short	Benchmark	17'645	12'685	--	--	--
	COT, no jumps	27'114	20'404	9'469	50.8%	50.8%
	COT, jumps	21'399	16'084	3'853	19.7%	18.9%
Medium	Benchmark	15'776	12'768	--	--	--
	COT, no jumps	22'400	19'589	6'623	37.1%	37.1%
	COT, jumps	17'366	15'280	2'231	11.8%	6.01%
Long	Benchmark	19'263	11'672	--	--	--
	COT, no jumps	25'102	16'589	5'839	27.9%	27.9%
	COT, jumps	18'141	12'140	1'901	10.5%	-7.87%

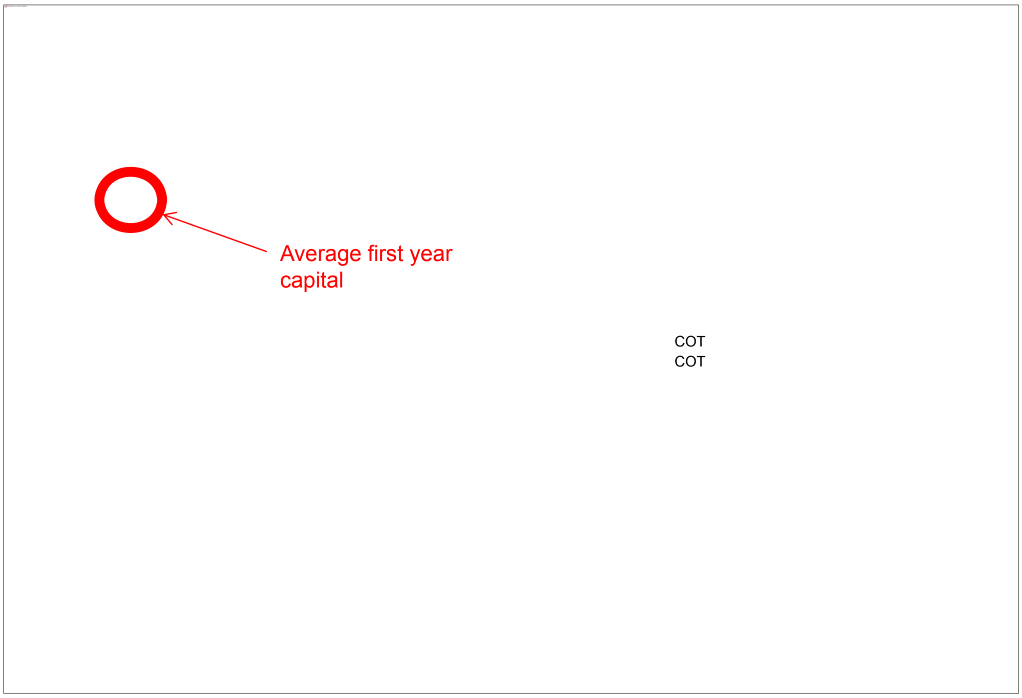
Capital Over Time in the Multiplicative Model (Short)

Average yearly capital requirement for the short-tailed pattern in proportion of the ultimate risk.



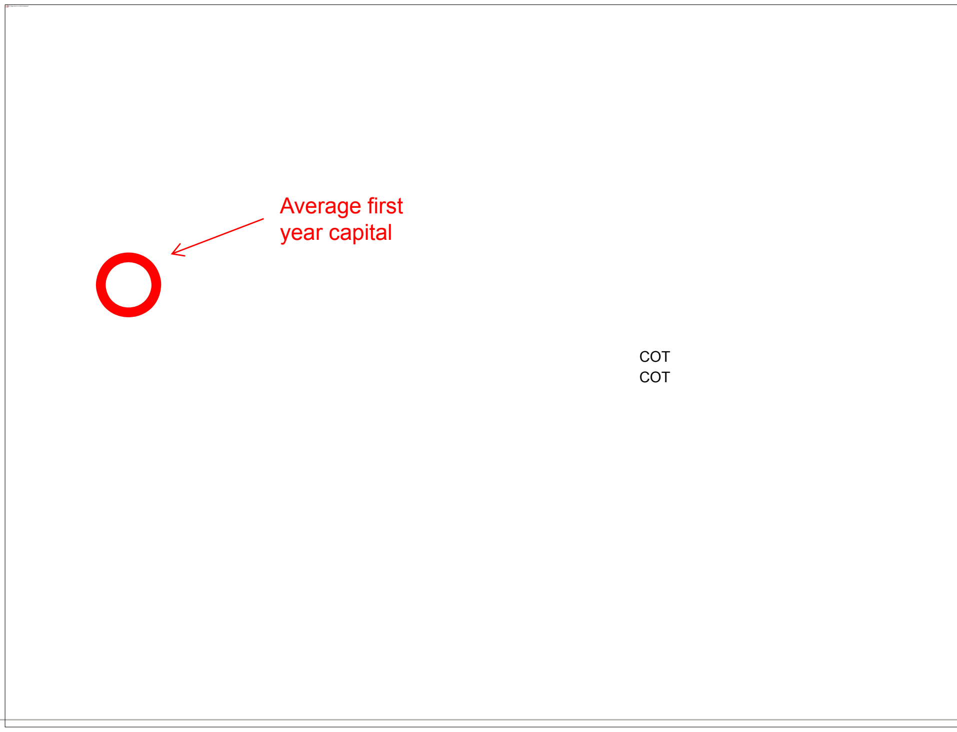
Capital Over Time in the Multiplicative Model (Medium)

Average yearly capital requirement for the medium-tailed pattern in proportion of the ultimate risk.



Capital Over Time in the Multiplicative Model (Long)

Average yearly capital requirement for the long-tailed pattern in proportion of the ultimate risk.



Agenda

1. Introduction
2. Current methods
3. A different approach to model the one year change
4. First year capital requirement comparison
5. Risk margin comparison
6. Conclusion

Conclusion

- We wanted to open up a new way of thinking for analyzing the one year change. This study shows that it is possible
- The idea is to naturally start from the beginning to reach the ultimate and to formulate a time-dependent process class for this
- We show that it is not only possible but it also gives us insight on the way other methods perform in describing the risk involved by such processes
- Misspecification for the Merz-Wüthrich method is dangerous. It can induce very large errors
- The COT method with jumps gives very good results for the medium and long-tailed log-normal triangles, but fair poorly to evaluate the risk margin for linear model. In the latter case, it estimates well the first year capital but strongly underestimates the required capital of later years
- For the COT method without jumps, it is the opposite. It estimates quite well the risk margin and yearly capitals in general for the linear model but overestimates the risk margin for log-normal triangles

The COT Method is a Good Approximation

- The COT method without jumps is more appropriate to model the reality described by the linear process, whereas the COT method with jumps is more appropriate to model the reality described by the multiplicative model. A (non-formal) mean diagonal correlation calculation justifies this result. Indeed, we obtain the following results for the mean correlation between the loss increments of two consecutive diagonals

Model	Mean diagonal correlation
Linear model	5%
Multiplicative model (Short)	55%
Multiplicative model (Medium)	57%
Multiplicative model (Long)	48%

Further Conclusions

- In all our cases, the COT method proves relatively efficient to calculate the one year capital. One should, however, handle this statement carefully as the method was used in a case where we have an efficient way of calculating the ultimate risk, which is not always guaranteed

- Overall, there is a real need to understand much more accurately the type of processes that drive the *development of insurance risks* and not only the ultimate risk

- This would bring closer together the P&C and Life approach to model risk

- Another difficulty arises in this case: even if a process describes well this development, there is no guaranty that it will work well should the legislation change

- In any case, the *time component of the process* can no more be ignored both for the solvency capital requirement and the risk margin