Performance evaluation of optimized portfolio insurance strategies (Joint work with: D. Zieling, S. Balder)

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1. Motivation and Outline
    - CPPI and recent developments
    - Outline of the further talk

CPPI and recent developments

- Constant proportion portfolio insurance (CPPI)
  - **Protection without options**
  - Dynamic portfolio of **underlying** and **risk–free asset**
  - **Cushion** C **management technique**
  - Cushion = difference between **portfolio value** V and **floor** F
  - **Floor** is defined by the **guarantee scheme** (e.g. simple floor growing with risk free rate or drawdown constraints)

Exposure E in the risky asset

$$E = \text{multiplier} \times \text{cushion} = m \times C$$

Recent developments in (C)PPI investments

- Variable multiples
  - Products allow for the multiple to vary over time in relation to the volatility of the risky asset

Advantages (disadvantages) of (C)PPI method

Portfolio insurance:

→ PI investor must give up upward participation to achieve the downward protection

Advantages of (C)PPI method

→ Simple investment rule, easy to explain to the customer

→ (C)PPI can be applied to an infinite investment horizon

→ Robustness (**model risk**): No gap risk within class of stochastic volatility models

Disadvantage of (C)PPI (complete market!)

→ Investor gives up more upward participation than OBPI investor

→ Put option is cheaper than zero bond (**kinked vs smooth solution**)

Outline

→ Performance evaluation of CPPI and variable multiplier strategies

Outline

- → Theoretical foundation and implementation of strategies
  - → Expected cushion growth rate maximizing (stochastic volatility setup)
  - → Assumption on risk premium: CPPI or variable multiplier
- → S&P 500 index return (and interest rate) data for 1985–2012
  - → Data and yearly evaluation of strategies (descriptive results)
- → Simulation model (EGARCH model, bootstrap on the residuals)
- → Simulation setup accounting of
  - → Transaction costs (trigger trading) and
  - → Gap risk
- → Conclusion and outlook

2 Strategies, data, and descriptive results

Strategies – Theoretical foundation

Model setup – stochastic volatility

→ Price dynamics of underlying

$$dS_t = S_t(\mu_t \, dt + \sigma_t \, dW_t^S)$$

→ W^S is one dimensional Brownian motion

→ σ_t is diffusion driven by W^σ

→ W^S and W^σ may be correlated

Proportional portfolio insurance (PPI) strategy ...

... with multiplier m_t

→ at t, m_t times the cushion is invested in the stock S

→ $(1 - m_t)$ is invested in the bank account B where

$$dB_t = B_t r_t \, dt$$

Optimization criterion

Cushion dynamics

$$dC_t = C_t \left(m_t \frac{dS_t}{S_t} + (1 - m_t) \frac{dB_t}{B_t} \right)$$
$$= C_t \left((r_t + m_t \lambda_t)\, dt + m_t \sigma_t \, dW_t^S \right)$$

→ $\lambda_t = \mu_t - r_t$ denotes the equity risk premium

Optimization criterion

→ Maximize expected cushion growth rate

$$E\left[\frac{1}{T} \ln \frac{C_T}{C_0} \right] = \frac{1}{T} E\left[\int_0^T \left(r_u + m_u \lambda_u - \frac{1}{2} (m_u \sigma_u)^2 \right) du \right]$$

Optimal strategy/multiplier

→ No inter–temporal hedging demand

→ For all $t \in [0, T]$, the optimal multiplier $m_t^{*,sv}$ is given by the optimal multiplier of an investor with a very short investment horizon, i.e.

$$m_t^{*,sv} = \operatorname{argmax}_{m_t} \left[m_t\lambda_t - \frac{1}{2}(m_t\sigma_t)^2 \right] = \frac{\mu_t - r_t}{\sigma_t^2} = \frac{\lambda_t}{\sigma_t^2}$$

Remark

→ Perspective of asset manager (index product, not individual (C)PPI)

→ Index products are based on TIPP (drawdown constraints)

→ We use a simple floor (initial floor F_0 which is then growing with r)

→ Qualitatively, evaluation of TIPP strategies gives same results (**but**, interpretation of cushion is difficult)

Optimal strategy and assumptions on equity risk premium

Optimal strategy and assumptions on equity risk premium

Assumption risk premium λ	optimal multiple m^*
(A0) $\lambda_t = \bar{\lambda}\sigma_t^2$ SR increasing in volatility	$m_t^* = \bar{\lambda}$ **constant**
(A1) $\lambda_t = \bar{\lambda}\sigma_t$ SR constant	$m_t^* = \bar{\lambda}\ \frac{1}{\sigma_t}$ prop. to inverse of local volatility
(A2) $\lambda_t = \bar{\lambda}$ RP constant	$m_t^* = \bar{\lambda}\ \frac{1}{\sigma_t^2}$ prop. to inverse of local variance

Strategies – Implementation

Implementation

→ Strategies are implemented in discrete–time (daily rebalancing)

→ Now, $t = 0, 1, 2, \ldots$ denote daily trading dates

→ $\hat{\lambda}$ and $\hat{\sigma}$ are long term (daily) estimates

→ $\hat{\sigma}_t = \sigma_{t,x\mathrm{M}}$ denote daily volatility estimates using a window of x months

→ We compare

- → **time–varying multiple strategies** m_t where

$$m_{t,(1),x\mathrm{M}} = \frac{\hat{\lambda}}{\hat{\sigma}} \frac{1}{\sigma_{t,x\mathrm{M}}}, \quad m_{t,(2),x\mathrm{M}} = \hat{\lambda} \frac{1}{\sigma^2_{t,x\mathrm{M}}}$$

- → and **optimal CPPI** $m^{*,\mathrm{const}} = \frac{\hat{\lambda}}{\hat{\sigma}^2}$

Variable multiplier strategies – Summary

Strategy	Variable m_t proportional to the inverse of the:
$m_{t,(1),1M}$	standard deviation of the latest 1 month (21 days) historical returns $(t,\ t-1, ..., t-20)$
$m_{t,(2),1M}$	variance of the latest 1 month (21 days) historical returns
$m_{t,(1),2M}$	standard deviation of the latest 2 month (42 days) historical returns $(t,\ t-1, ..., t-41)$
$m_{t,(2),2M}$	variance of the latest 2 month (42 days) historical returns
$m_{t,GARCH}$	one day ahead variance (forecast) based on simulation model implied $\sigma^2_{t+1,GARCH}$

Return data (S&P500 – price index)

Return data (S&P500 – price index)

- → Bloomberg data for the time period 1985–2012
  - → Daily simple returns
  - → Number of observation 7,044
- → Interest rate data
  - → Discount yields of T-Bills (91 days to maturity)

→ Based on daily simple excess returns, we consider the yearly outcomes of PPI strategies

Summary and test statistics of daily (yearly) returns (whole data)

Mean excess return	0.000201	(0.053716)	
Standard deviation	0.011677	(0.188286)	
Skewness	-0.843287		
Kurtosis	24.749300		
Minimum	-0.204590		
Maximum	0.115778		

	t-statistic	critical value ($\alpha = 0.1\%$)	p-value
Skewness	-28.8982	-3.29	0.0000
Kurtosis	745.3170	3.29	0.0000
Normality (Jarque-Bera)	139,958	14.67	0.0000

$\rightarrow$ Significant negative skewness

$\rightarrow$ Significant excess kurtosis

Descriptive results – Yearly performance of past 27 years

Panel A: Unbounded investment quote (no borrowing constraints)

	$\frac{1}{T}E[\ln \frac{C_T}{C_0}]$	$\frac{1}{T}E[\ln \frac{V_T}{V_0}]$	$E[V_T]$	min V_T
$m_{t,(1),1M}$	0.065	0.045	106.099	**79.752**
	0.318	*0.167*	*19.127*	
$m_{t,(2),1M}$	**0.074**	**0.079**	**121.852**	72.317
	0.624	*0.397*	*98.088*	
$m = 1$	0.030	0.018	102.157	81.567
	0.167	*0.081*	*7.992*	
$m^{*,\text{const}}$	0.031	0.023	103.057	73.894
$=1.4741$	*0.255*	*0.120*	*11.804*	
$m = 2$	0.023	0.027	103.962	66.808
	0.358	*0.163*	*15.940*	
$m = 4$	-0.116	0.019	106.569	52.766
	0.851	*0.310*	*31.199*	

→ Initial investment $V_0 = 100$, guarantee/floor $F = 50$

→ *Standard deviation* in italics

Observations

- → min V_T is higher than guarantee ($F = 50$) (**no gap risk**)
- → Worst case (min $V_T = 52.77$) is linked to the CPPI with $m = 4$
  - → Although average value of $m_{t,(2),1\mathrm{M}}$ is 3.94 ($m_{t,(2),1\mathrm{M}}$ varies between 0.0587 and 25.84)
- → Among the CPPI strategies, $m^{*,\mathrm{const}}$ gives highest average cushion growth rate
- → But, **time–varying multiples** give better results
  - → $m_{t,(1),1\mathrm{M}}$ yields a 110% higher average cushion growth rate
  - → $m_{t,(2),1\mathrm{M}}$ even a 130% higher average cushion growth rate
- → Growth rates of leveraged strategies are highly volatile
- → None of the comparative growth rate results is significant!

Additional performance measures

→ Consider additional performance measures

Additional performance measures – Summary

Sharpe ratio (SR)	$\frac{E\left[V_T - V_0 e^{rT}\right]}{\sqrt{\mathrm{Var}[V_T]}}$
Adjusted Sharpe ratio ($ASSR$)	$SR\sqrt{1 + b_3 \frac{\mathrm{Skew}}{3} SR}$ where $b_3 = 2$
Omega measure (Ω)	$\frac{E[\max\{V_T - K, 0\}]}{E[\max\{K - V_T, 0\}]}$
Sortino ratio (SoR)	$\frac{E[V_T - K]}{\sqrt{E[(\max\{K - V_T, 0\})^2]}}$
Upside potential ratio (UPR)	$\frac{E[\max\{V_T - K, 0\}]}{\sqrt{E[(\max\{K - V_T, 0\})^2]}}$

Descriptive results – additional performance measures

Panel A: Unbounded investment quote (no borrowing constraints)

	SR	$ASSR$	$\Omega - 1$	SoR	UPR
$m_{t,(1),1M}$	**0.319**	0.644	1.553	0.798	1.313
$m_{t,(2),1M}$	0.223	**0.709**	**3.104**	**1.835**	**2.426**
$m = 1$	0.270	–	0.966	0.427	0.870
$m^{*,\text{const}}$	0.259	–	0.904	0.416	0.877
$m = 2$	0.249	0.082	0.847	0.407	0.888
$m = 4$	0.211	0.241	0.691	0.375	0.916

Observations

- → Sharpe ratio (*SR*) is mean–variance–based
  - → If investor values skewness positively, *SR* overrates strategies reducing skewness (value strategies) and underrates momentum (PI) strategies
  - → For CPPI strategies, *SR* is the lower the higher the leverage is
  - → Ranking of PI strategies with *SR* is not meaningful here
- → Adjusted for skewness Sharpe ratio (*ASSR*) and other performance measures
  - → Better performance of the time–varying multiple strategies compared to the optimal constant multiple strategy

Problem

→ **Variable multipliers are promising candidates** to outperform (CPPI) strategies (w.r.t. the expected (cushion) growth rates and other performance measures)

Problem

→ Yearly non–overlapping historical return blocks do not allow the deduction of any significant performance results

→ Leveraged strategies imply volatile terminal values

→ Sufficiently high number of observations (daily return paths) needed

→ Construct simulation model which mimics the empirical return distributions as close as possible

4 Simulation tool

- Construction of simulation tool
- Estimated model

→ Student's t–EGARCH model to describe the data

→ Conditional (log) variance model combined with MA(2) conditional mean model for excess returns R_t is

EGARCH model

$$\text{MA(2)}: \; R_t = \theta_0 + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \epsilon_t, \qquad \epsilon_t = \sigma_t z_t$$

$$\text{EGARCH}(P,Q): \; \ln \sigma_t^2 = \omega + \sum_{j=1}^{P} \left[\alpha_j \left(|z_{t-j}| - E[|z_{t-j}|]\right) + \gamma_j z_{t-j}\right] + \sum_{k=1}^{Q} \beta_k \ln \sigma_{t-k}^2$$

where $E[|z_{t-j}|] = E\left[\frac{|\epsilon_{t-j}|}{\sigma_{t-j}}\right] = \sqrt{\frac{\nu-2}{\pi}} \frac{\Gamma((\nu-1)/2)}{\Gamma(\nu/2)}$ for $z_t \sim T(\nu)$

→ $\Gamma(x)$ denotes the gamma function

→ $T(\nu)$ a Student's t distribution with $\nu > 2$ degrees of freedom

Parameter estimates

Parameter estimates for the MA(2)-*t*-EGARCH(1,1) model

Parameter	Value	Standard Error	t-statistic
θ_0	0.000201	–	–
θ_1	-0.013733	0.0122840	-1.1180
θ_2	-0.019380	0.0116200	-1.6678
ω	-0.106670	0.0151860	-7.0240
α_1	0.112720	0.0099974	11.2747
β_1	0.988490	0.0016127	612.9561
γ_1	-0.084188	0.0071361	-11.7976
ν	5.700800	0.3786400	15.0558

$\rightarrow$ θ_0 is fixed to mean of empirical excess return series

We compare

- → Model specified by the estimated parameters ($z_t \sim T(\nu)$ with $\nu = 5.7008$)
- → Semiparametric model with z_t drawn randomly from the set of empirical residuals $\hat{z}_{emp} = \{\hat{z}_t\}$

Summary statistics of empirical and simulated daily excess returns

Returns	Mean	Stdev.	Skewness	Kurtosis	Min	Max
empirical	0.00020	0.0117	-0.8433	24.7493	-0.2046	0.1158
$z_t \sim T(\nu)$	0.00020	0.0113	0.0054	14.0842	-0.6270	0.8704
$z_t \sim \hat{z}_{emp}$	0.00020	0.0117	-0.8190	20.6388	-0.6987	0.3629

- → Both simulated series represent 200,000 years, each with 260 daily excess returns

Simulation results ...

... are stated in two parts

→ First part: Without transaction costs

- → Distribution of variable multipliers
- → Daily changes of variable multipliers
- → Key numbers characterizing the turnovers

→ Second part: Proportional transaction costs

- → Trigger trading (stochastic trading dates)
- → Reconsider the evaluation of the PPI strategies

Summary statistics of variable multipliers m_t and the relative changes $\Delta m_t = \Delta m_t = \left|\frac{m_t - m_{t-1}}{m_{t-1}}\right|$

	Mean	Median	Stdev	Skewness	Kurtosis	Min	Max
$m_{t,\text{GARCH}}$	3.16	2.47	**2.55**	2.30	12.17	0.01	46.18
$m_{t,(1),1M}$	2.19	2.00	1.04	1.31	6.17	0.12	16.11
$m_{t,(2),1M}$	4.09	2.79	**4.35**	4.01	36.87	0.01	180.46
$m_{t,(1),2M}$	2.09	1.95	0.91	1.11	5.14	0.16	12.54
$m_{t,(2),2M}$	3.62	2.63	3.42	3.12	21.26	0.02	109.37
$\Delta m_{t,\text{GARCH}}$	**0.08**	0.07	0.07	3.32	19.94	0	0.83
$\Delta m_{t,(1),1M}$	0.04	0.02	0.06	4.12	32.95	0	2.11
$\Delta m_{t,(2),1M}$	**0.08**	0.04	0.12	5.78	83.43	0	8.66
$\Delta m_{t,(1),2M}$	0.02	0.01	0.03	5.00	48.71	0	1.25
$\Delta m_{t,(2),2M}$	0.04	0.02	0.06	5.47	68.98	0	4.08

→ Results are based on a simulation of 50,000 years, each with 260 trading days

Observations

- → $m_{t,\text{GARCH}}$ is, on average, 3.158
  - → More than two times higher than $m^{*,\text{const}} = 1.4741$
- → Multiplier based on 1M estimation horizon, $m_{t,(2),1M}$, is even more extreme
  - → It ranges from slightly above zero to a 180.462
  - → Distribution is positively skewed and exhibits high kurtosis (36.866)
- → Variable multiplier based on the inverse of the volatility, $m_{t,(1)}$, possesses a sample distribution with a skewness of 1.31 (1.11) and a kurtosis of 6.17 (5.14) for the 1M (2M) estimation window

→ **High standard deviations and high average percentage changes** for multiples **proportional to the inverse of the variance**

Turnovers – Key numbers

→ High turnovers indicate that the performance may deteriorate under transaction costs

→ Based on relative daily turnovers δ_t^S

$$\delta_t^S := \frac{\left| m_t C_t - m_{t-1} C_{t-1} \frac{S_t}{S_{t-1}} \right|}{V_t}, \text{ we consider}$$

Key numbers

→ **Maximum relative daily turnovers**
$Maxturn = E[\max_{t \in \{1,\ldots n-1\}} \delta_t^S]$

→ **Expected total relative turnovers**
$Totturn = E[\sum_{t=1}^{n-1} \delta_t^S]$

→ **Expected number of trading days per year**
$Trades = E[\sum_{t=0}^{n-1} 1_{\delta_t^S > 0}]$

Performance results for dynamic and constant multiplier strategies (based on $M = 50{,}000$ simulations)

	$E[\ln \frac{C_T}{C_0}]$	$E[\ln \frac{V_T}{V0}]$	$E[V_T]$	$\min\ V_T$	*Maxturn*	*Totturn*
$m_{t,\text{GARCH}}$	0.070	0.050	106.924	65.609	**0.669**	**15.333**
$m_{t,(1),1\text{M}}$	0.065	0.045	105.931	63.134	0.443	9.661
$m_{t,(2),1\text{M}}$	0.065	0.051	107.233	56.206	**0.718**	**12.552**
$m_{t,(1),2\text{M}}$	0.063	0.044	105.743	64.162	0.259	5.333
$m_{t,(2),2\text{M}}$	0.064	0.050	107.057	59.512	0.450	7.385
$m = 1$	0.036	0.022	102.648	54.916	0.000	0.000
$m^{*,\text{const}}$	0.042	0.031	103.925	50.893	0.019	0.679
$m = 2$	(0.039); **1**	0.038	105.344	48.664	0.049	1.903
$m = 4$	(-0.051); **41**	0.048	109.877	43.244	0.219	6.435

Observations

- → Basically, descriptive results are confirmed
- → But, the simulation model also accounts of gap risk
  - → Guarantee violations (gap events) for $m = 2$ and $m = 4$
  - → No gap events for variable multiplier strategies
- → Out–performance of time–varying multiplier strategies is valid (robust w.r.t. all performance measures except SR)
  - → Surprisingly, feasible variable multiplier strategies perform quite similarly
  - → Universally, their performance results are rather close to the optimal result obtained by the variance estimate which is based on the simulation model
  - → But, the time–varying multiple strategies afford high turnovers

→ **Reconsider performance evaluation accounting of transaction costs with adequate trigger**

Transaction Costs

High turnovers

- → Accounting of transaction costs is important for PPI strategies
- → Strategies imply a reduction (increase) of the asset exposure in falling (rising) markets
- → Investor suffers from any round–turn in the asset prices
- → Effect is severe if there are in addition transaction costs

- → We consider proportional transaction costs denoted by a proportionality factor θ
- → For daily trading, the cushion dynamics are the

$$C_{t+} = C_t - \theta \left| m_t C_{t+} - m_{t-1} C_{(t-1)+} \frac{S_t}{S_{t-1}} \right| ,$$

where $m_t C_{t+}$ denotes the asset exposure immediately after a transaction cost adjustment

Trigger Trading

Trigger design

- τ is sequence of stopping times where $\tau_k \in \{0, 1, \ldots, n-1\}$, $\tau_0 = 0$ and $\tau_{k+1} > \tau_k$
- Assume that $C_{\tau_k +} > 0$
  - Number of risky assets (constantly held immediately after τ_k) is $\eta_{\tau_k +} = \frac{m_{\tau_k} C_{\tau_k +}}{S_{\tau_k}}$
  - Implicit multiplier at t $(\tau_{k+} < t < \tau_{k+1})$ is

$$m_t^{imp} = \frac{\eta_{\tau_k +} S_t}{C_t}$$

  - Target multiplier m_t is defined by PPI rule
  - Trigger design with trigger level φ is

$$\tau_k := \inf \left\{ t > \tau_{k-1} \middle| \{m_t^{imp} \leq \frac{1}{\varphi} m_t\} \cup \{m_t^{imp} \geq \varphi\, m_t\} \right\}$$

→ Each strategy is evaluated w.r.t. its (expected cushion growth rate) optimal trigger level φ^*

Observations (prop. transaction costs with $\theta = 0.1\%$)

- → **CPPI strategies**
  - → Optimal level $\varphi^*(m)$ is the higher, the higher the multiplier is
  - → Optimized trigger levels are close to one (close to daily trading)
- → **Variable multiple strategies**
  - → Optimized trigger levels range from $\varphi^* = 1.2$ for $m_{t,(1),2M}$ to $\varphi^* = 2.0$ for $m_{t,\text{GARCH}}$
  - → Optimal trigger level is the higher, the higher the dispersion of the multiplier values is

→ Corresponding trigger levels are omitted in the following

Performance results for dynamic and constant multiplier strategies under transaction costs and optimized trigger trading

	$E[\ln \frac{C_T}{C_0}]$	$E[\ln \frac{V_T}{V0}]$	$E[V_T]$	min V_T	*Maxturn*	*Totturn*
$m_{t,\text{GARCH}}$	0.059	0.045	106.329	62.978	0.999	3.331
$m_{t,(1),1\text{M}}$	0.056	0.042	105.682	61.857	0.717	2.378
$m_{t,(2),1\text{M}}$	0.051	0.046	106.851	55.033	1.007	4.530
$m_{t,(1),2\text{M}}$	0.056	0.040	105.431	63.170	0.369	2.057
$m_{t,(2),2\text{M}}$	0.053	0.045	106.675	57.164	0.692	3.079
$m = 1$	0.035	0.022	102.596	54.911	0.000	0.000
$m^{*,\text{const}}$	0.040	0.029	103.795	50.627	0.027	0.036
$m = 2$	(0.037); **1**	0.036	105.172	48.512	0.102	0.260
$m = 4$	(-0.059); **51**	0.046	109.644	26.377	0.365	1.790

Observations (prop. transaction costs with $\theta = 0.1\%$)

The trigger design implies

(i) Fewer trades (small deviations of target multiple m_t and implied multiple m_t^{imp} are not taken into account)

(ii) Less turnovers in total

→ Expected cumulated turnovers *Totturn* are reduced by more than 50% compared to daily trading

(iii) Average and maximum turnover volume per trade *Maxturn* increase

→ In practice, maximal turnovers which are above 50% are often considered as prohibitive

→ Highest average turnovers are ca 100% for $m_{t,\mathrm{GARCH}}$ and $m_{t,(2),1M}$

→ Drawback of a time–varying multiple proportional to the inverse of the estimated variance

6 Conclusion and outlook

- Conclusion

Conclusion

- → **Industry's approach (rule based and variable multiplier)**
  - → In line with well known optimization problems
  - → Multipliers based on rolling window of historical volatilities significantly outperform CPPI
    (the result is robust w.r.t. alternative performance measures)
  - → Performance of a multiplier prop. to the inverse of the variance is slightly better than using the inverse of the volatility
  - → **But**, accounting of maximal per annum turnovers are in favor of a multiple **proportional to the inverse of the one day ahead volatility** (industry's approach)
- → Additional good news
  - → Proportionality to the inverse of the volatility also reduces gap risk