HEDGING SYSTEMATIC MORTALITY RISK WITH MORTALITY DERIVATIVES

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OUTLINE

- Introduction
  - Mortality risk management
  - Valuation
- Mortality risk
  - Unsystematic risk
  - Systematic risk
- Hedging mortality risk
  - Survivors swaps
  - A modeling framework
- Illiquid assets
  - Risk-minimization
  - Inherent risk and survivor swaps

RISK MANAGEMENT IN LIFE INSURANCE

Current situation: Market based valuation of assets and liabilities

Financial risks: Partly controlled via investments, derivatives etc

Mortality and longevity risk:

An essential **non-hedged** and **non-hedgeable** (?) risk for pension funds

Solvency II: Capital requirements derived from properties for actual products, inherent risks and investments

Capital requirements and risk margins for life annuities is to be calculated using a 20 percent reduction in mortality rates

Mortality derivatives could lead to lower capital requirements!
(e.g. survivor swaps, survivor bonds, other constructions)

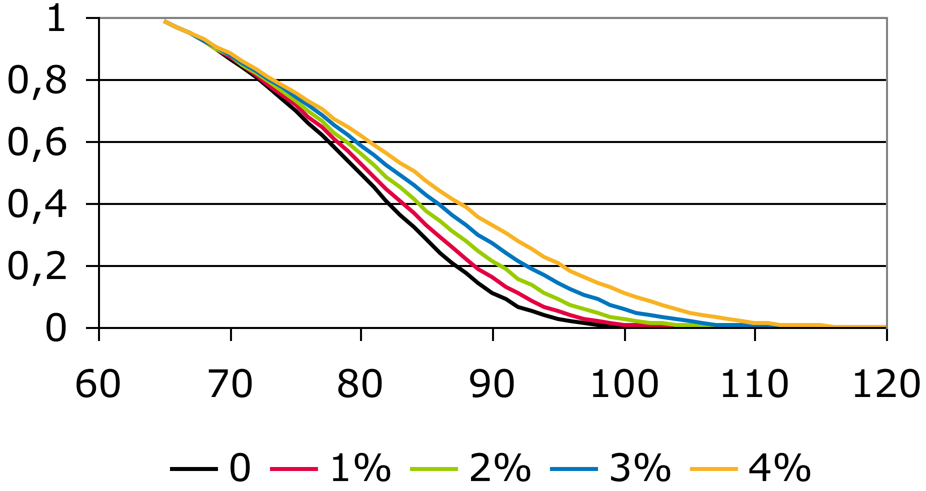
VALUATION AND MORTALITY RISK MANAGEMENT

Key issues

- Expected future mortality development (**trend**, **volatility**)
- Level of risk premium
- Natural buyers and sellers of mortality risk

Illustration of importance of trend for valuation

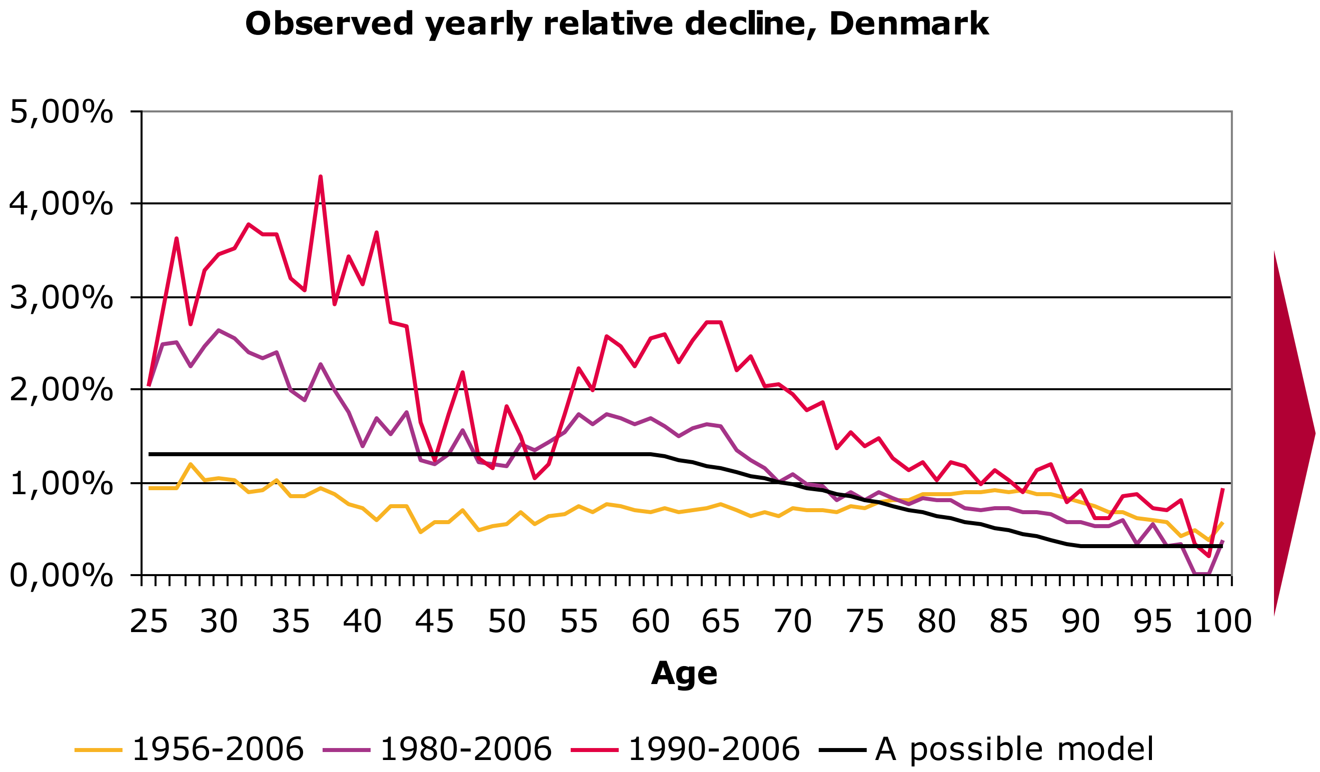
Survival probability (initial age 65)



Example with ”old” mortality:

Trend	V	E	$_{25}p_{65}$
0%	10.66	80.3	12.5%
1%	11.04	81.2	17.3%
2%	11.46	82.3	22.7%
3%	11.92	83.6	28.4%
4%	12.43	85.1	34.3%

IMPROVEMENT RATES FOR DIFFERENT PERIODS



Difficult to predict future changes…

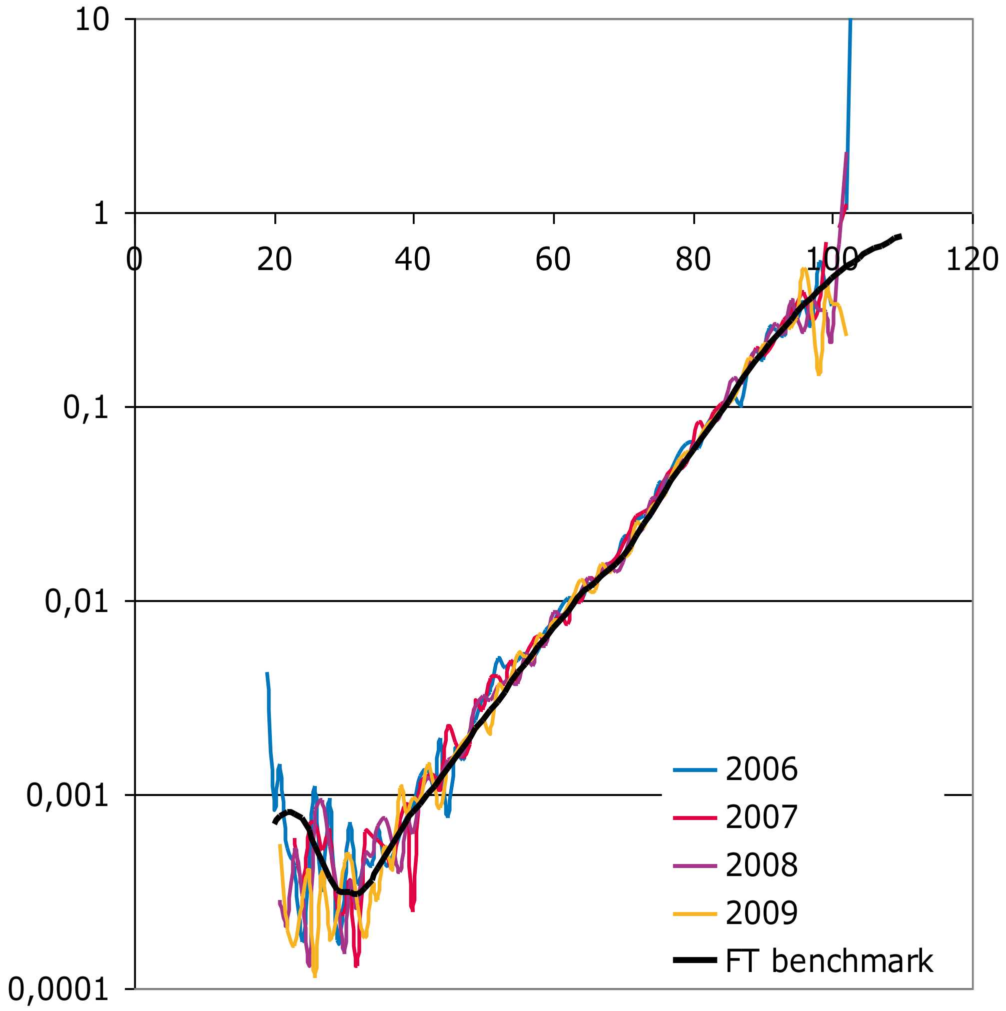
Yearly average improvement rates depend on the period considered.
Largest improvement rates observed in the period 1990-2006

Source: PFA Pension & Human mortality database, www.mortality.org

APPROACH TO LONGEVITY MODELING IN DENMARK

- Danish FSA has introduced a **mortality and longevity benchmark**
- Benchmark for **current mortality** estimated from data for insured individuals with 5 years of data
- Pension funds perform yearly statistical tests whether they derive from benchmark
- Benchmark for **future trend** estimated from total Danish population (20 years of data)

BENCHMARK - OE-RATES 2006-2009 - MALES



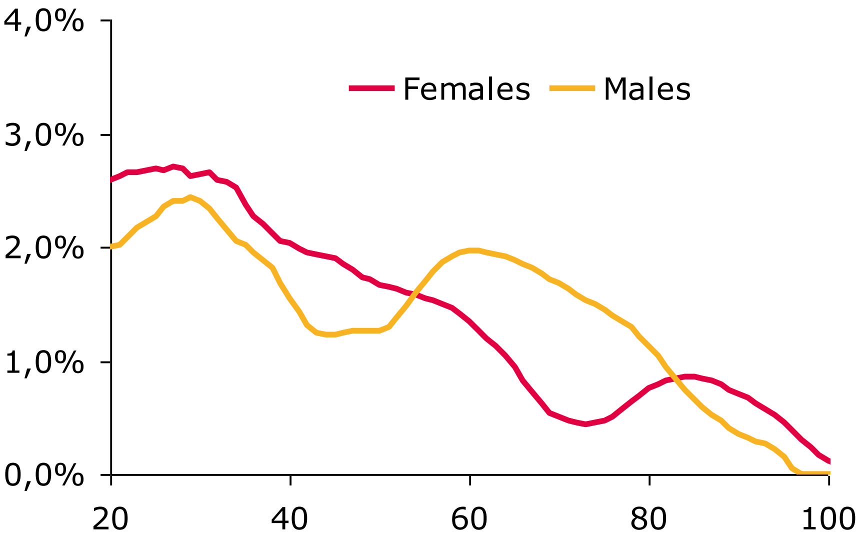
- Benchmark determined using splines from (log) linear regression
- Special model for high age mortality - logistic model
- ”Kannisto model” suggested by Thatcher et al, 1998
- Not exponential increase of mortality after age 100

APPROACH TO LONGEVITY MODELING IN DENMARK

Benchmark

- Future trend assumption based on observations for 20 years
- Age-dependent yearly decline
- Different decline rates for males and females

- Trend and current mortality lead to **best estimate**
- **Solvency 2:** 20 % reduction of mortality rates



Age	Males	Females
40	84,0	86,5
50	83,5	86,2
60	83,8	86,6
65	84,3	87,0
70	85,1	87,7
80	88,1	90,0
90	93,9	94,7

TWO TYPES OF MORTALITY RISK

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TWO FUNDAMENTALLY DIFFERENT TYPES OF MORTALITY RISK

Systematic mortality risk:

- Unexpected changes in underlying mortality intensities and expected life times:
  - Same effect on all individuals ⇨ **NOT diversifiable**
  - Long-term risk
  - Risk management solutions: new products, mortality derivatives

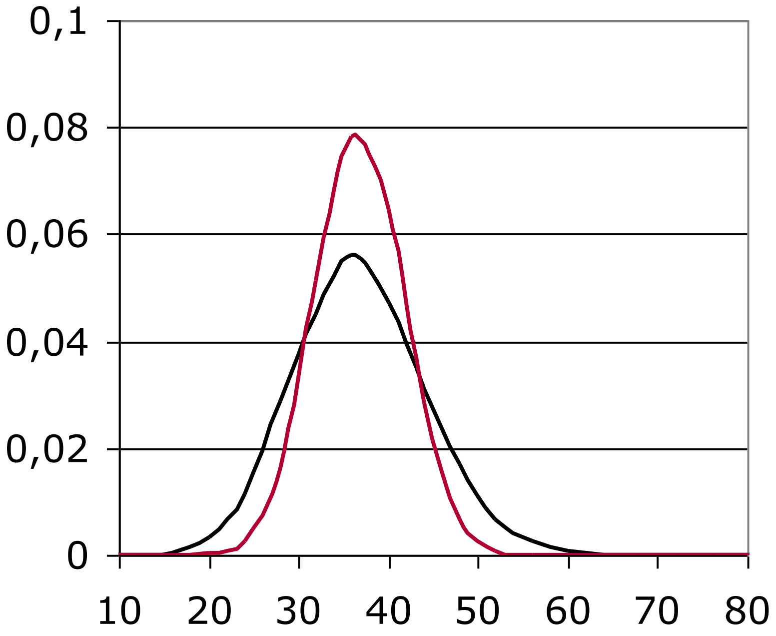
Interaction is not trivial!

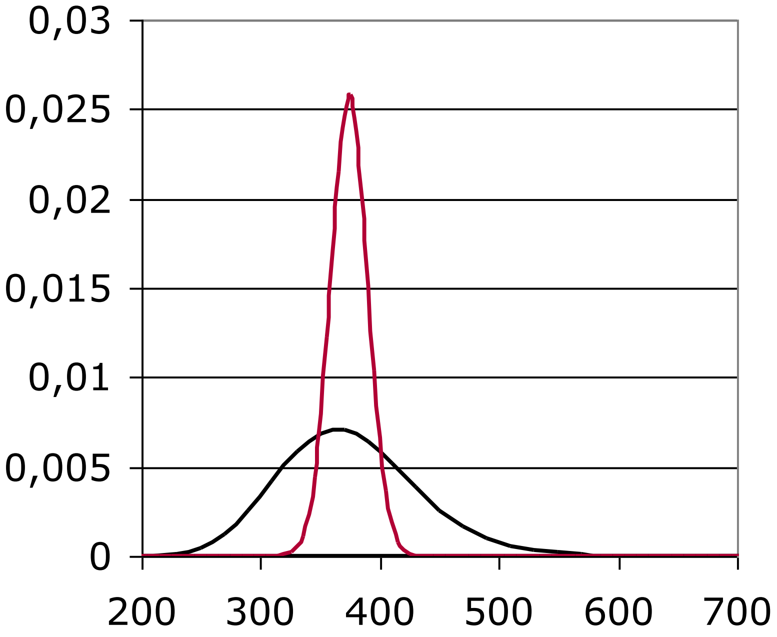
Unsystematic mortality risk:

- Randomness of deaths given underlying intensities:
  - Law of large numbers (Jakob Bernoulli, 1654-1705)
  - Risk is **diversifiable**
  - Short and long term risk

LONG TERM SIMULATION OF NUMBER OF SURVIVORS

Simulated number of survivors at age 85, given initial age 30

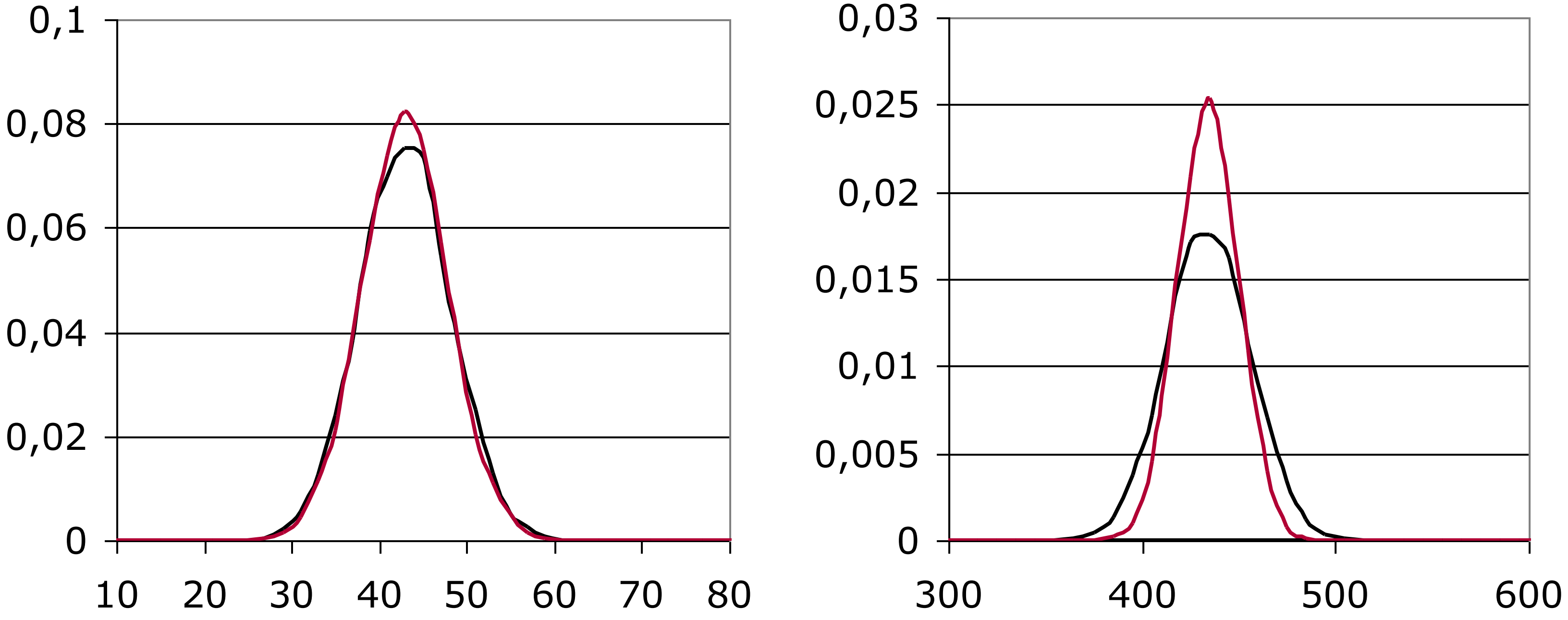




100 individuals	**1000 individuals**
Expected number of survivors: 37	Expected number of survivors: 374
Std. dev. without systematic risk: **4.9**	Std. dev. without systematic risk: **15.4**
Std. dev. with systematic risk: **7.3**	Std. dev. with systematic risk: **57.7**

SHORT TERM SIMULATION OF NUMBER OF SURVIVORS - PORTFOLIO OF RETIRED

Simulated number of survivors at age 85, given initial age 75



100 individuals	1000 individuals
Expected number of survivors: 43.5	Expected number of survivors: 435
Std. dev. without systematic risk: **4.9**	Std. dev. without systematic risk: **15.8**
Std. dev. with systematic risk: **5.2**	Std. dev. with systematic risk: **21.5**

SURVIVOR SWAPS AND MORTALITY RISK MODELING

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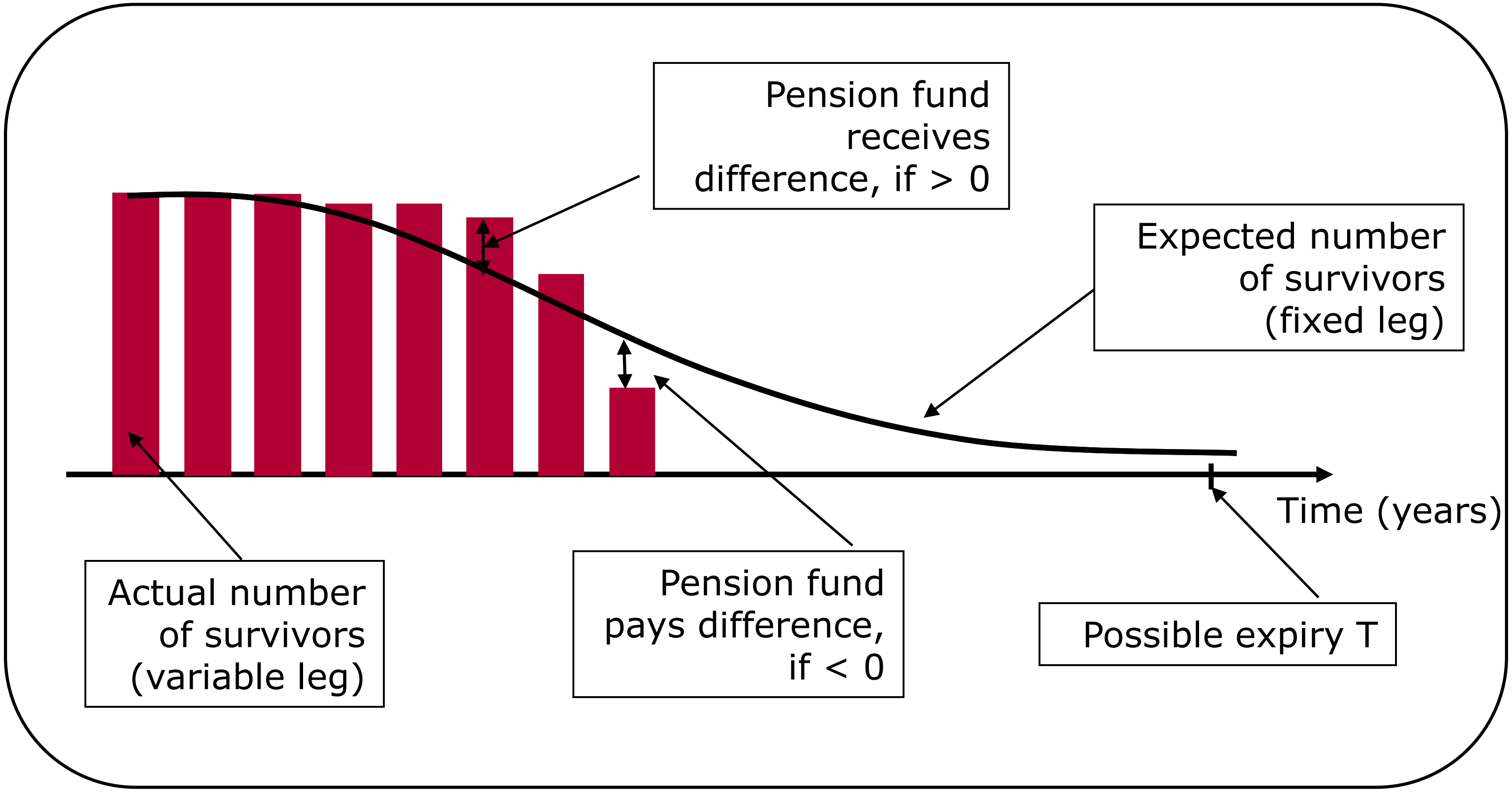
SURVIVOR SWAPS

Fix a portfolio of (insured) lives:



”Expected” number may include a risk premium

SURVIVOR SWAP – UNDERLYING PORTFOLIO

Possibilities

Own portfolio

Other "small" portfolio

Large reference population

Choice of underlying portfolio:

- Survivor swap linked to own portfolio may provide "perfect hedge" but may be less liquid **(no market)**
- Using a "small" portfolio on other lives will typically not provide a good hedge. Unsystematic risk will dominate **(investment instrument)**
- Large reference population gives desired properties if sufficiently long time horizon **(more liquid, possible market?)**

SURVIVOR BONDS

Y (t,x): Number of persons alive at t, aged x at time 0

Survivor index: S(t,x) = Y(t,x)/Y(0,x)

(observable, stochastic process, non-traded)

Payment process: dB(t) = S(t,x) dt, t < T

Cannot be replicated with existing instruments!

Expected relative number of survivors: $E_Q[S(t,x)] = {}_tp^Q_x$(choice of Q?)

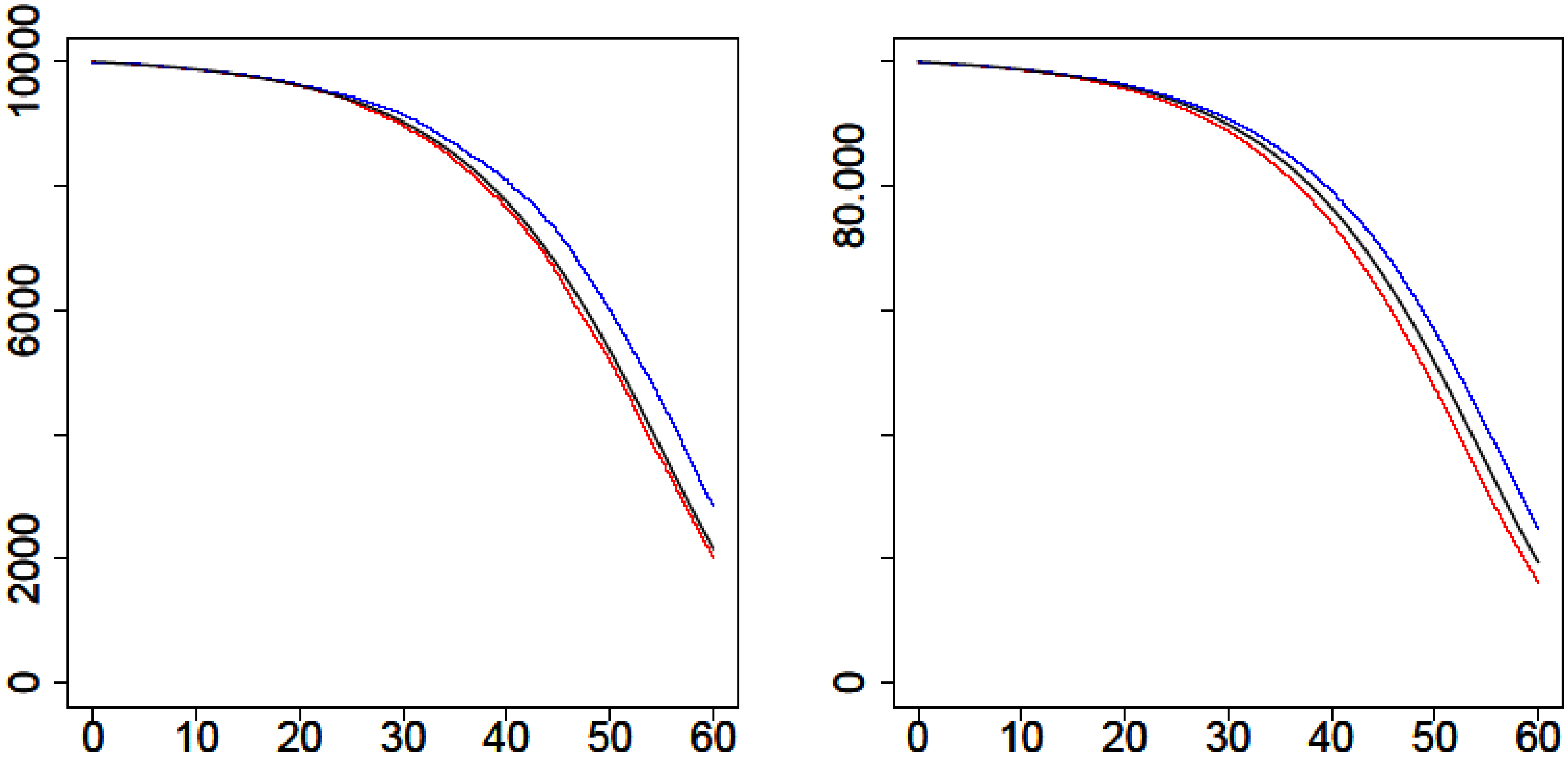
Price:

$$E^Q\left[\int_t^T e^{-\int_t^u r(s)ds} S(u,x)du \,\middle|\, F(t)\right] = \int_t^T P(t,u)E^Q\left[S(u,x)\middle|F(t)\right]du$$

Discounting factor - interest rate risk

longevity risk

SURVIVOR BONDS: EXAMPLE OF PAYMENTS



Actual number of survivors compared to expected number of survivors (black line) in two different stochastic scenarios (red and blue lines). Left plot shows number of survivors in the insurance portfolio and right plot shows number of survivors in larger portfolio. (Portfolio sizes 10.000 and 100.000)

SURVIVOR SWAPS

Payment process: $dB^{swap}(t) = (S(t,x) - {}_t p^{Q'}_x)dt, \; t < T$

"Variable" payments

"Fixed" payments

Survival probability chosen af time 0! Pricing:

$$\int_t^T P(t,u)E^Q\left[S(u,x)\middle|F(t)\right]du \quad - \quad \int_t^T P(t,u)\; {}_u p_x^{Q'} du$$

Valuation of variable payments:

- Similar to a **life annuity**
- Interest rate dependent
- Value reflects mortality development
- Increases if mortality decreases

Valuation of fixed payments:

- Similar to a **certain annuity**
- Interest rate dependent
- Price does not depend on mortality development

For pension fund:

Swap part of market-based accounting

Consistent market values for all assets and liabilities

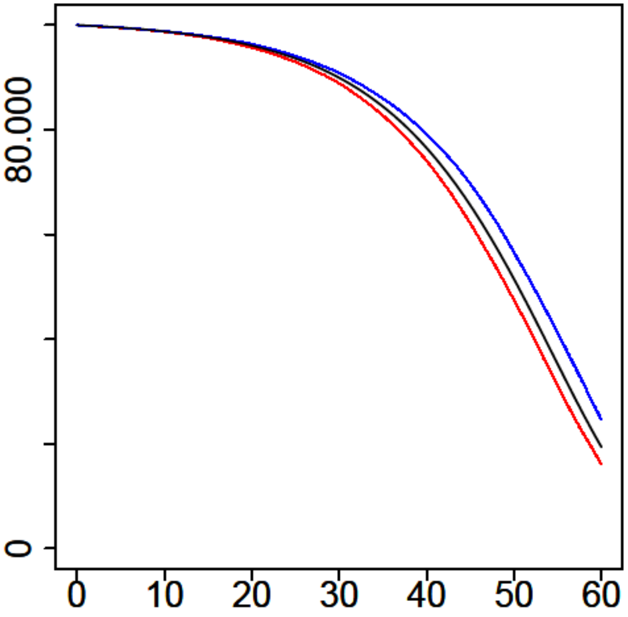
SURVIVOR SWAPS – ACTUARIAL OR FINANCIAL INSTRUMENT?

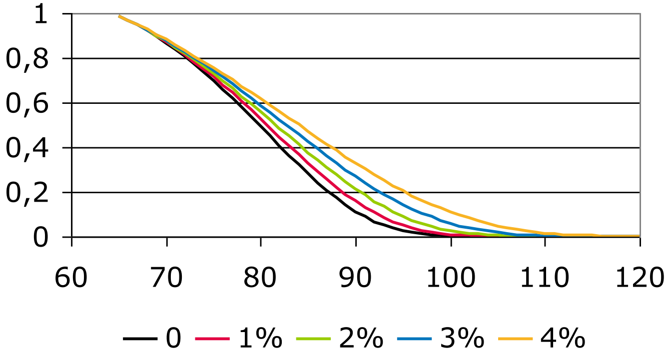
Actuarial interpretation – own portfolio

- Match payments from life annuities
- If more annuitants survive, pension fund receives the difference
- Not to be traded?
- Entered with reinsurance company

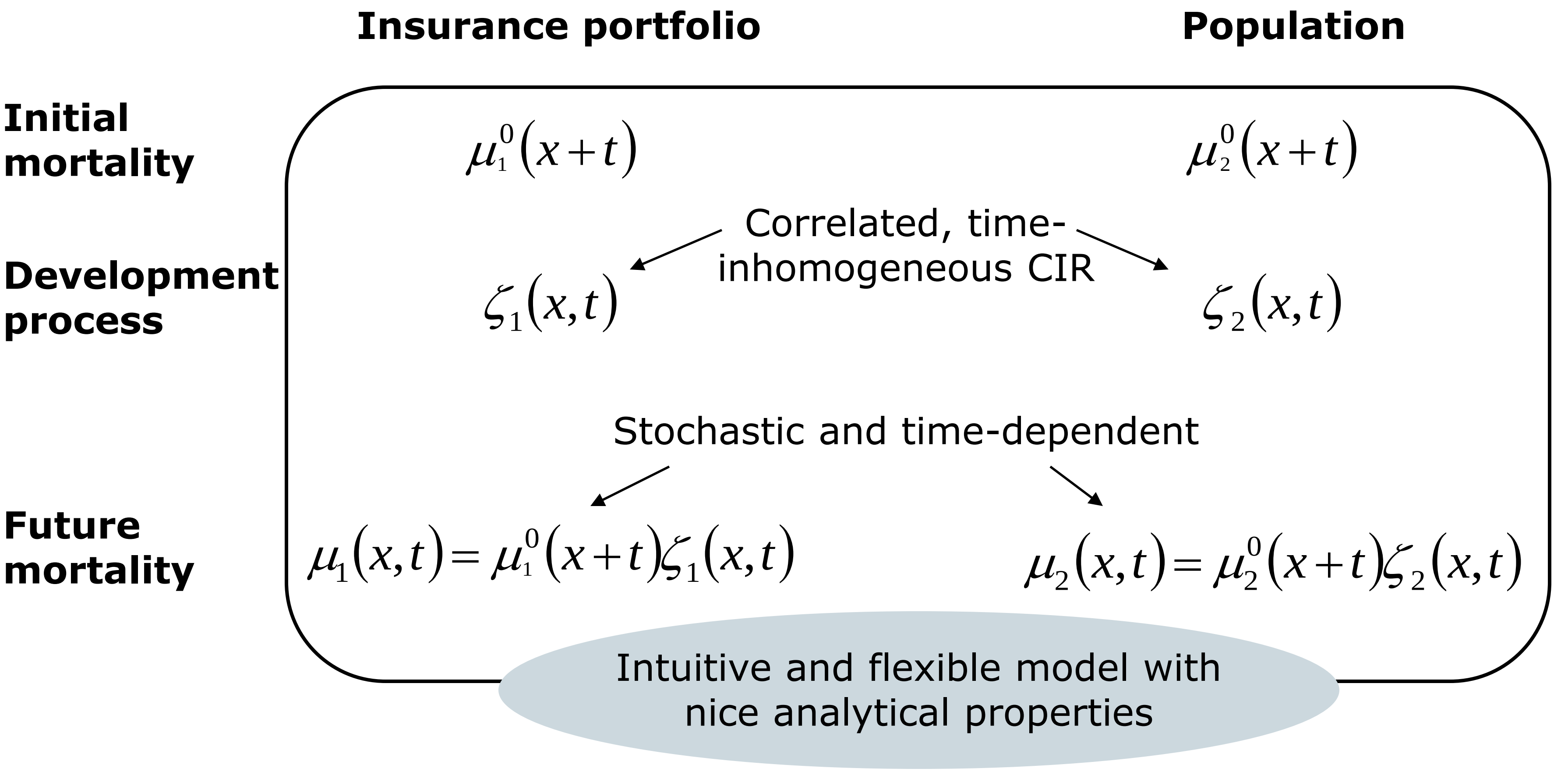
Financial interpretation

- Market value of future payments has similar sensitivity towards mortality/longevity risk as existing liabilities
- Value of survivor swap increases after a longevity stress (Solvency II) ⇨ **Capital relief**
- Trading possbilities?





MORTALITY MODEL AND PORTFOLIOS



Insurance portfolio | **Population**

Initial mortality

$\mu_1^0(x+t)$

$\mu_2^0(x+t)$

Development process

Correlated, time-inhomogeneous CIR

$\zeta_1(x,t)$

$\zeta_2(x,t)$

Future mortality

Stochastic and time-dependent

$\mu_1(x,t)=\mu_1^0(x+t)\zeta_1(x,t)$

$\mu_2(x,t)=\mu_2^0(x+t)\zeta_2(x,t)$

Intuitive and flexible model with nice analytical properties

NUMERICAL EXAMPLES

Time-inhomogeneous CIR model known from finance

$$d\zeta(x,t)=(\gamma(x,t)-\delta(x,t)\zeta(x,t))dt+\sigma(x,t)\sqrt{\zeta(x,t)}dW^{\mu}(t)$$

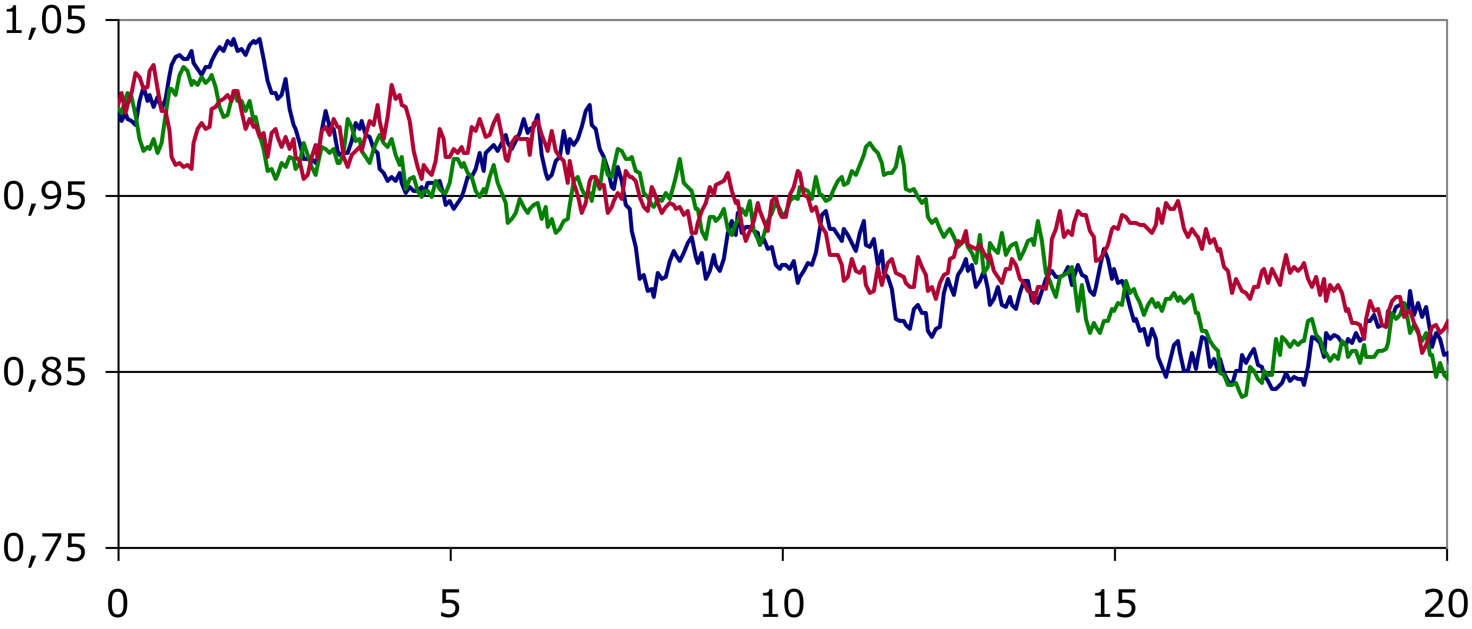
Parameterization

	$\delta(x,t)$	$\gamma(x,t)$	$\sigma(x,t)$
Case I	$\tilde{\delta}$	$\tilde{\delta}e^{-\tilde{\gamma}t}$	$\tilde{\sigma}$
Case II	$\tilde{\gamma}$	$\frac{1}{2}\tilde{\sigma}^2$	$\tilde{\sigma}$

Quantiles, time horizon 20 years:

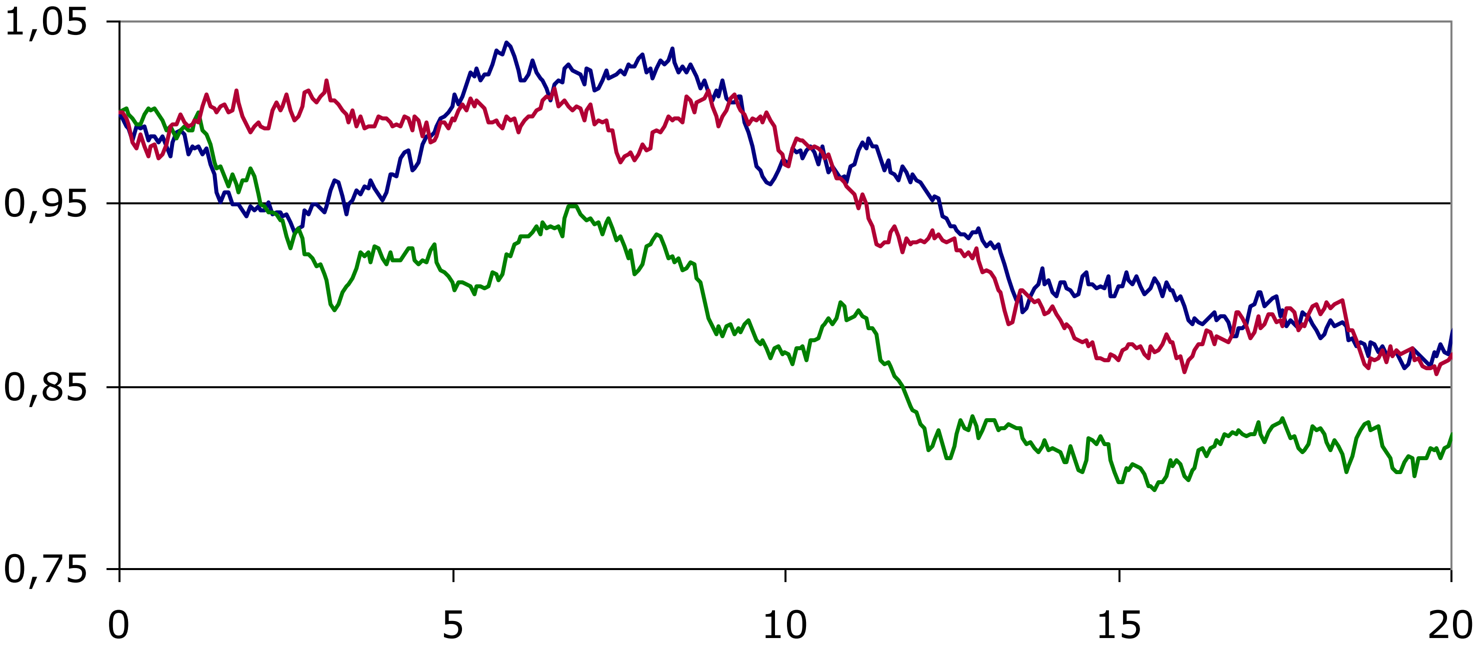
	$\tilde{\delta}$	$\tilde{\gamma}$	$\tilde{\sigma}$	5%	25%	50%	75%	95%
Case I	0.2	0.008	0.03	0.814	0.856	0.886	0.917	0.962
Case II		0.008	0.02	0.726	0.801	0.854	0.909	0.990

SIMULATION FOR IMPROVEMENT PROCESS (CASE I)



(with mean-reversion)

SIMULATION FOR IMPROVEMENT PROCESS (CASE II)



(without mean-reversion)

MIXED DYNAMIC AND STATIC HEDGING

DYNAMIC **RISK-MINIMIZATION** FOR PAYMENT STREAMS

Idea Minimize conditional expected (squared) in- or outflow not generated by *A using a market measure Q*

Method Market value decomposition

$$V^{*,Q}(t)=V^{*,Q}(0)+\int_0^t \xi^A(u)dX(u)+\int_0^t \vartheta^A(u)dY(u)+L^A(t)$$

Unhedgeable risk

Risky assets

Optimal strategy

$\xi^o(t)=\xi^A(t)$ ← Number of asset *X*

$\vartheta^o(t)=\vartheta^A(t)$ ← Number of asset *Y*

$\eta^o(t)=V^{*,Q}(t)-A^*(t)-\xi^A(t)X(t)-\vartheta^A(t)Y(t)$ ← Investment in bank account

Results are typically intuitive!

A NAIVE MIXED DYNAMIC AND STATIC RISK-MINIMIZING STRATEGY

Set-up with an illiquid asset

X still traded dynamically

Y is an illiquid asset traded at fixed times t_i only

A naive approach

$\hat{\xi}^o(t) = \xi^o(t)$ ⟵ Investment in *X* unchanged

$\hat{\vartheta}^o(t) = \vartheta^o(t_{i-1})$ for $t \in \left(t_{i-1}, t_i\right]$ ⟵ Investment in *Y* fixed at optimal investment at beginning of period

Does not work if *X* and *Y* are correlated or trend in $\vartheta^o(t)$

MIXED DYNAMIC AND STATIC RISK-MINIMIZING STRATEGY

Trick to handle correlation between *X* and *Y*

Decompose *Y* with respect to *X* $dY(t) = \xi^{Y}(t)dX(t) + dL^{Y}(t)$

Optimal strategy for $t \in (t_{i-1}, t_i]$

$$\hat{\xi}^{o}(t) = \xi^{o}(t) + \xi^{Y}(t)\left(\vartheta^{o}(t) - \hat{\vartheta}^{o}(t)\right)$$

Optimal dynamic strategy corrected by hedgeable part of illiquid asset

$$\hat{\vartheta}^{o}(t) = \frac{E^{Q}\left[\int_{t_{i-1}}^{t_i} \vartheta^{A}(u)dL^{Y}(u)\Delta L^{Y}(t_i) \middle| F(t_{i-1})\right]}{E^{Q}\left[\left(\Delta L^{Y}(t_i)\right)^{2} \middle| F(t_{i-1})\right]}$$

Risk adjusted average of dynamic strategy on next interval

$$\hat{\eta}^{o}(t) = V^{*,Q}(t) - A^{*}(t) - \hat{\xi}^{o}(t)X(t) - \hat{\vartheta}^{o}(t)Y(t)$$

CASE STUDY WITH SURVIVOR SWAPS

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Survivor swap payment process

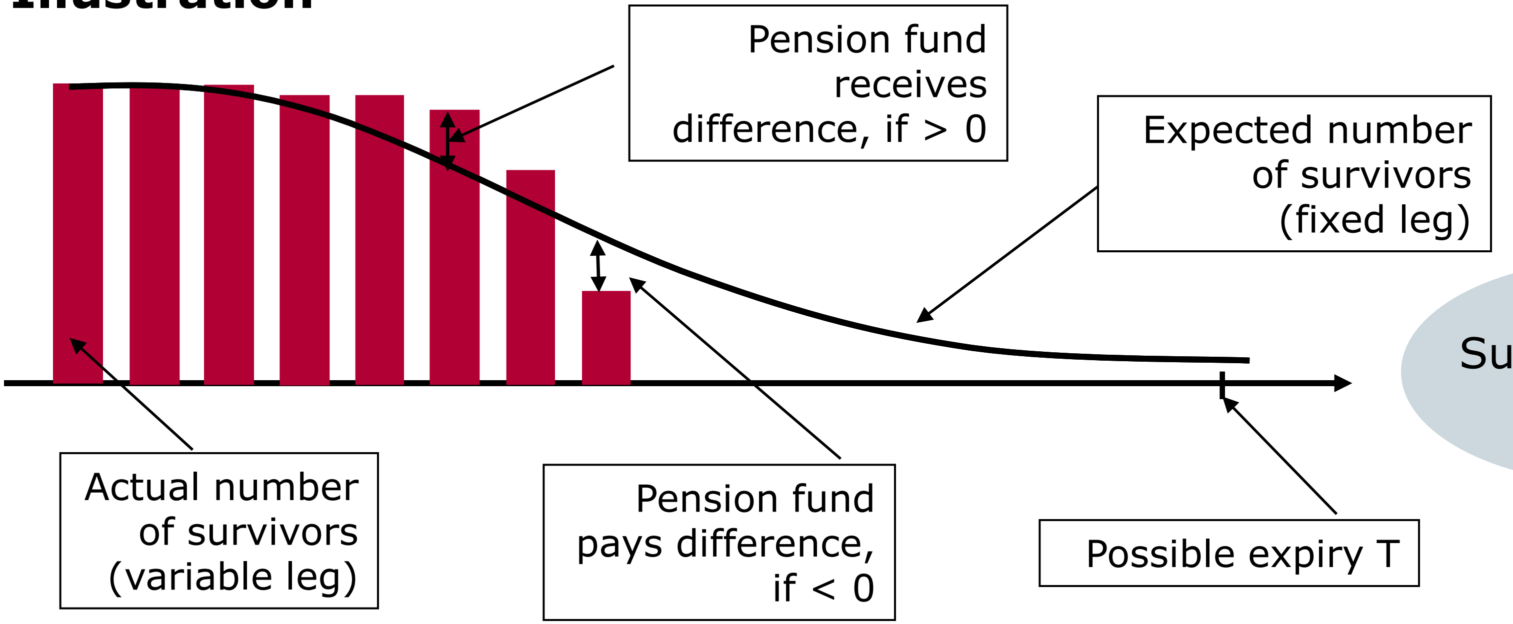
$$dA_j^{swap}(x,t) = \left(n_j - N_j(x,t)\right)dt - n_{j\,t}\,\tilde{p}_x\,dt$$

Number of deaths

Initial number of lives

Agreed survival probability

Illustration



Pension fund receives difference, if > 0

Expected number of survivors (fixed leg)

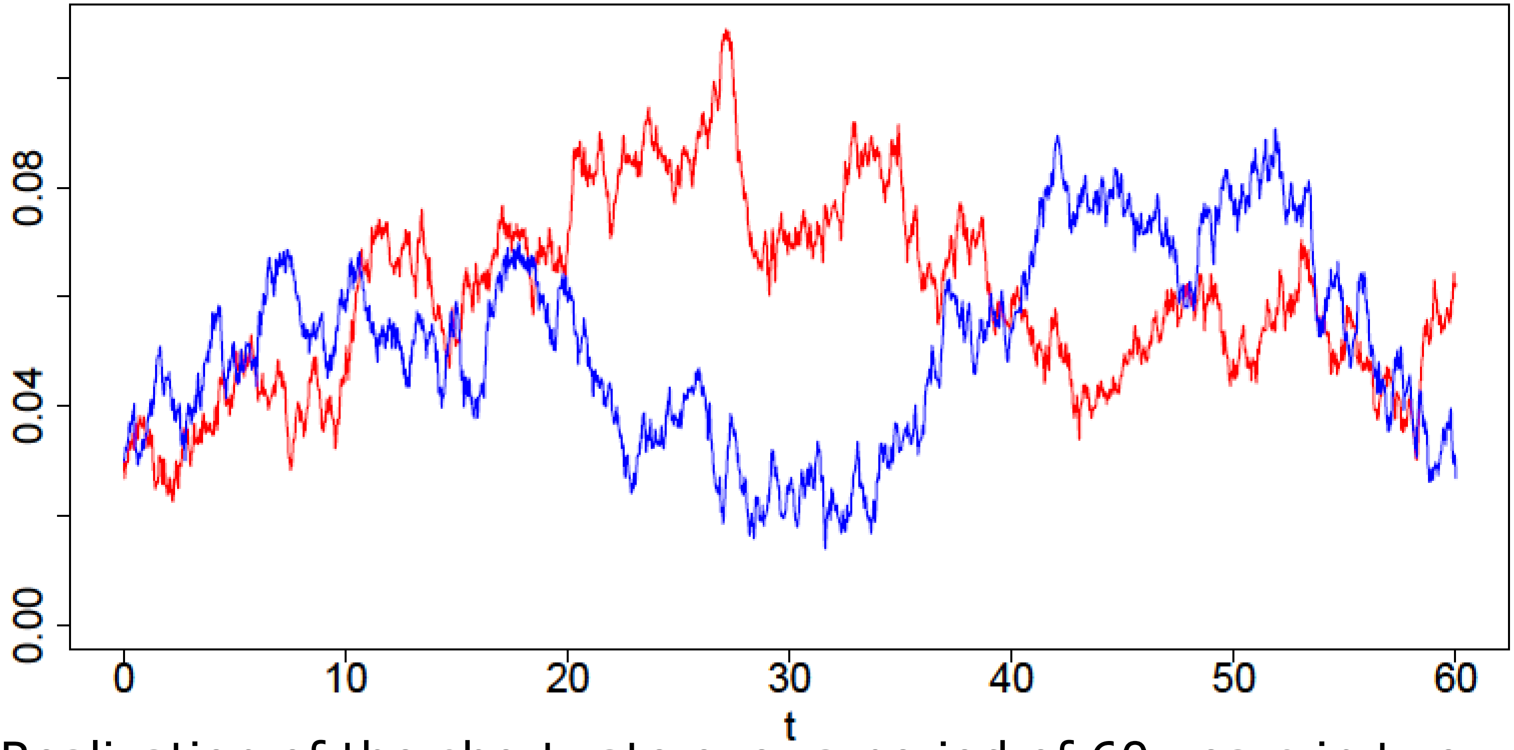
Actual number of survivors (variable leg)

Pension fund pays difference, if < 0

Possible expiry T

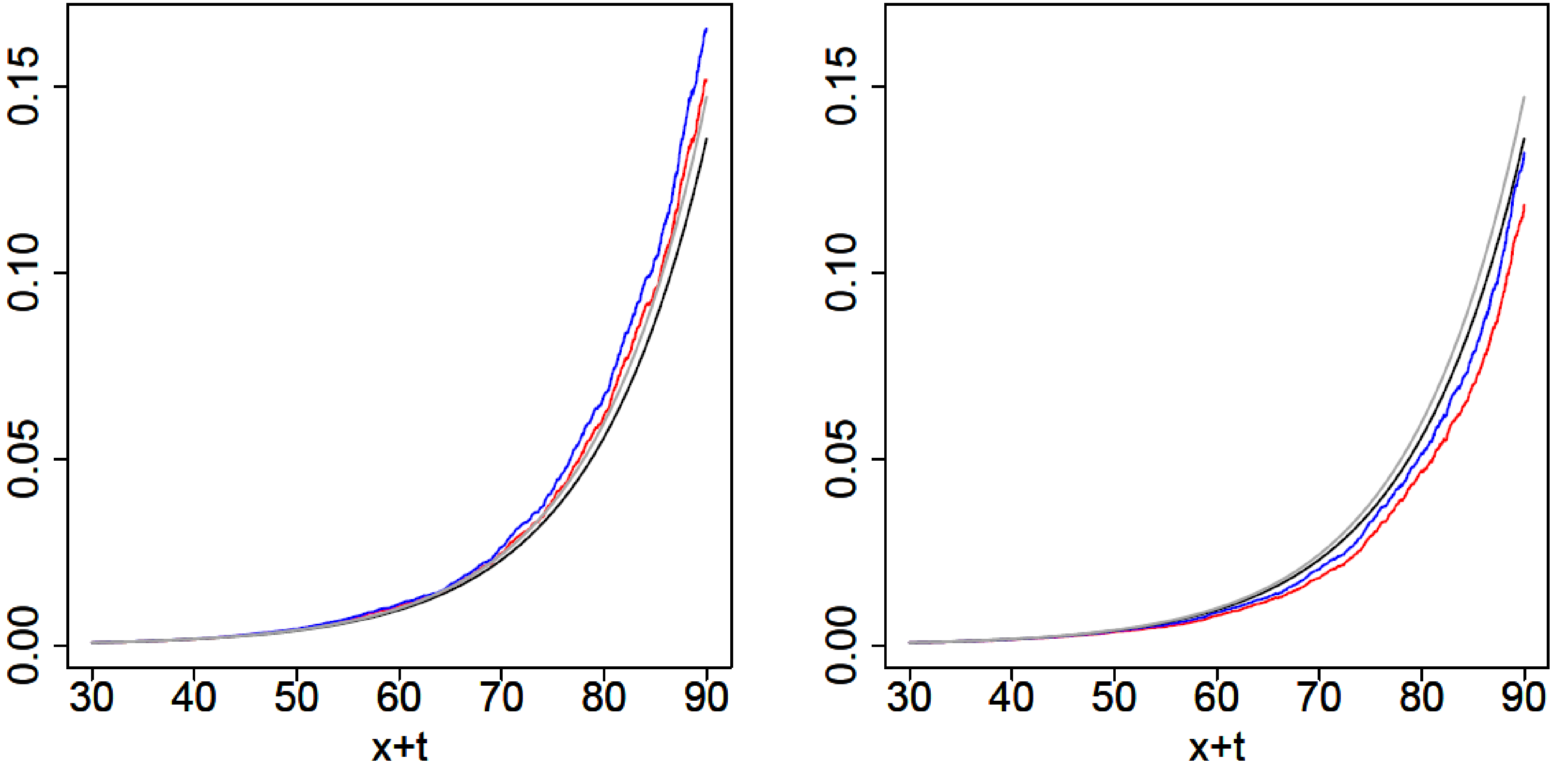
Survivor swaps are illiquid

SIMULATED INTEREST RATE SCENARIOS



Realization of the short rate over a period of 60 years in two difference stochastic scenarios

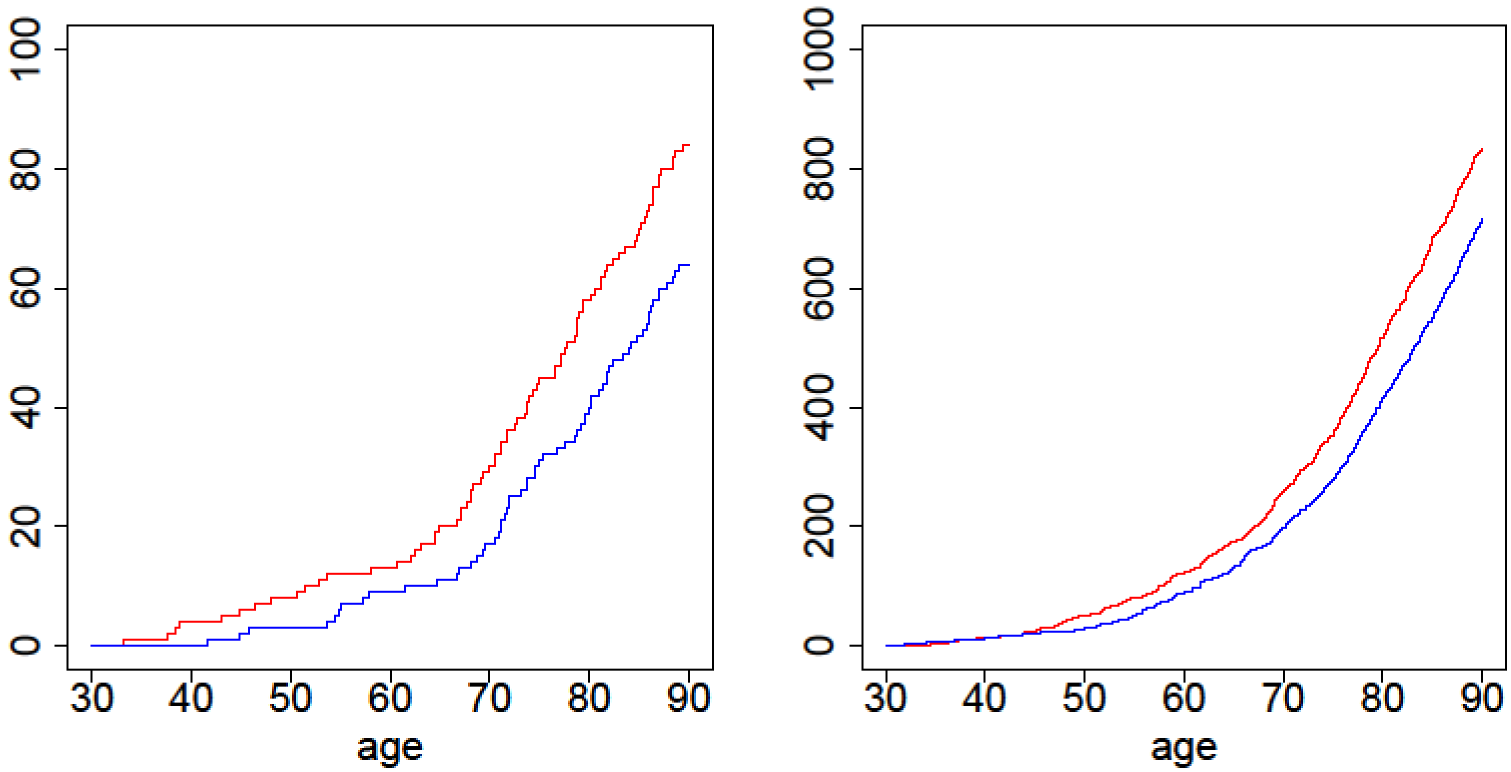
SIMULATED MORTALITY INTENSITIES



Mortality intensities for the insurance portfolio (red lines) and the population (blue lines) in two stochastic scenarios (left plot and right plot).

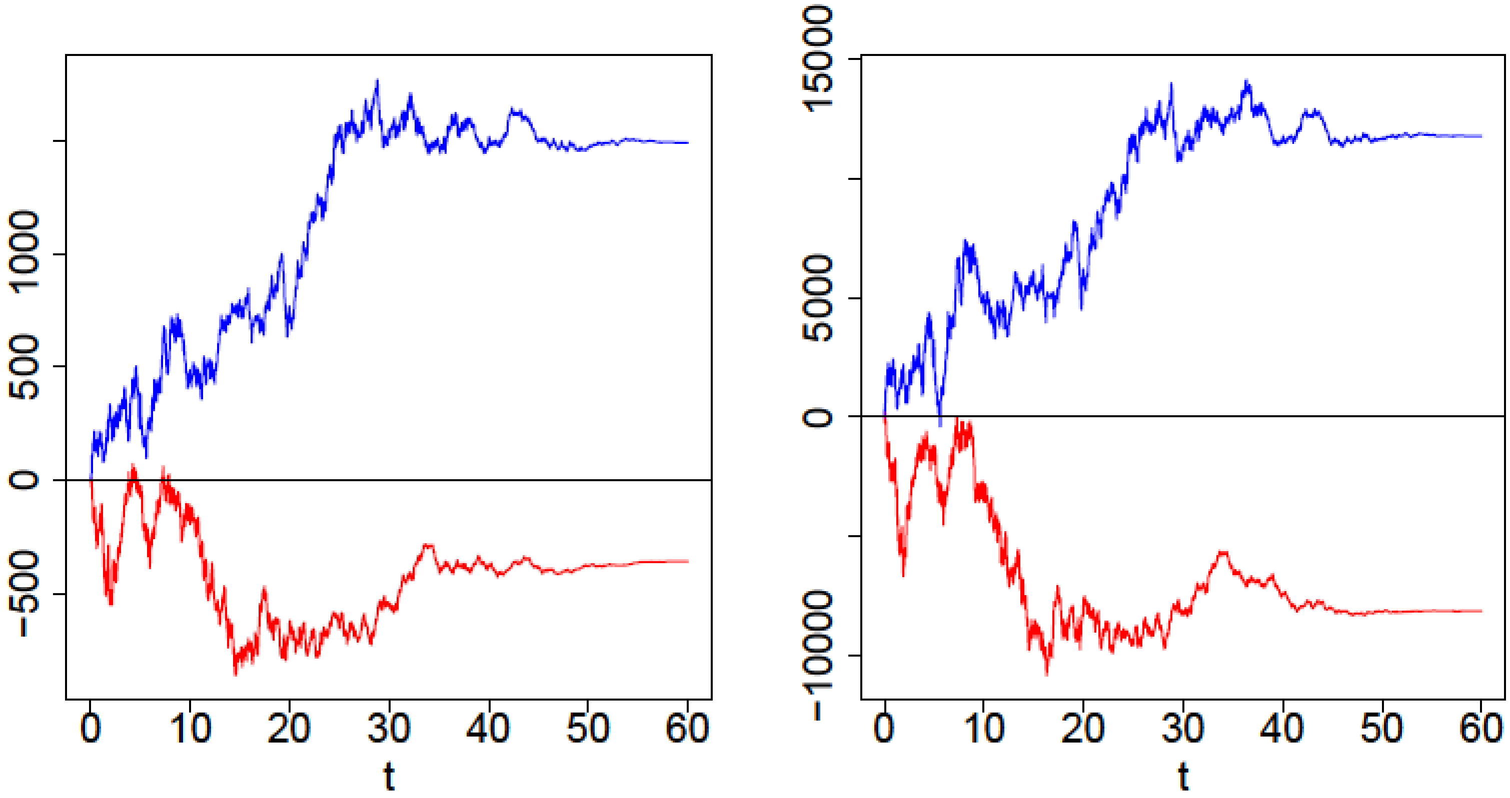
Deterministic mortality intensities with a trend of decline (black and grey lines)

SIMULATED DEATHS IN THE TWO SCENARIOS



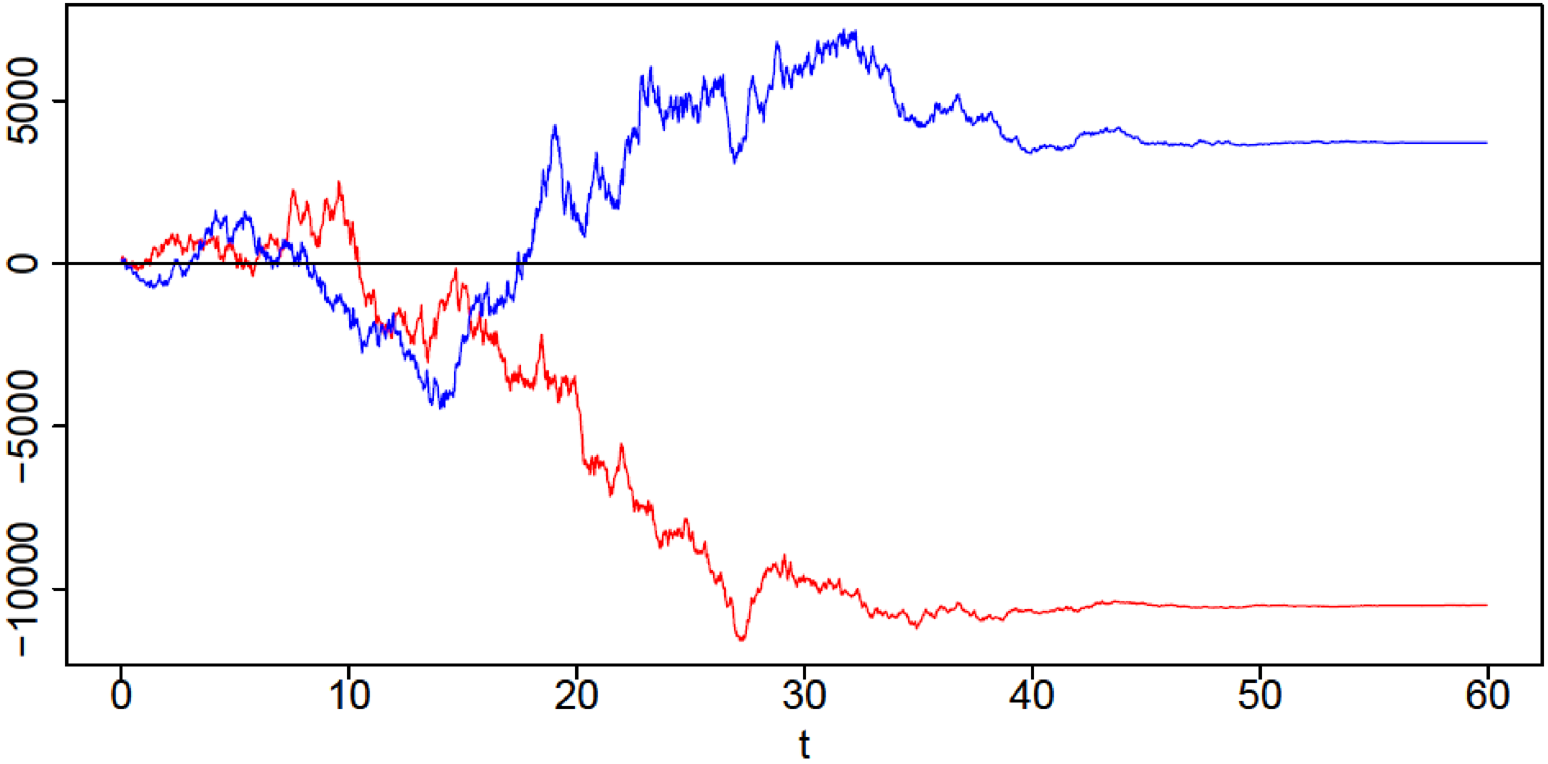
Deaths in the insurance portfolio (left plot) and deaths in the population (right plot) in the two scenarios

SIMULATED SURVIVOR SWAP PRICE PROCESSES



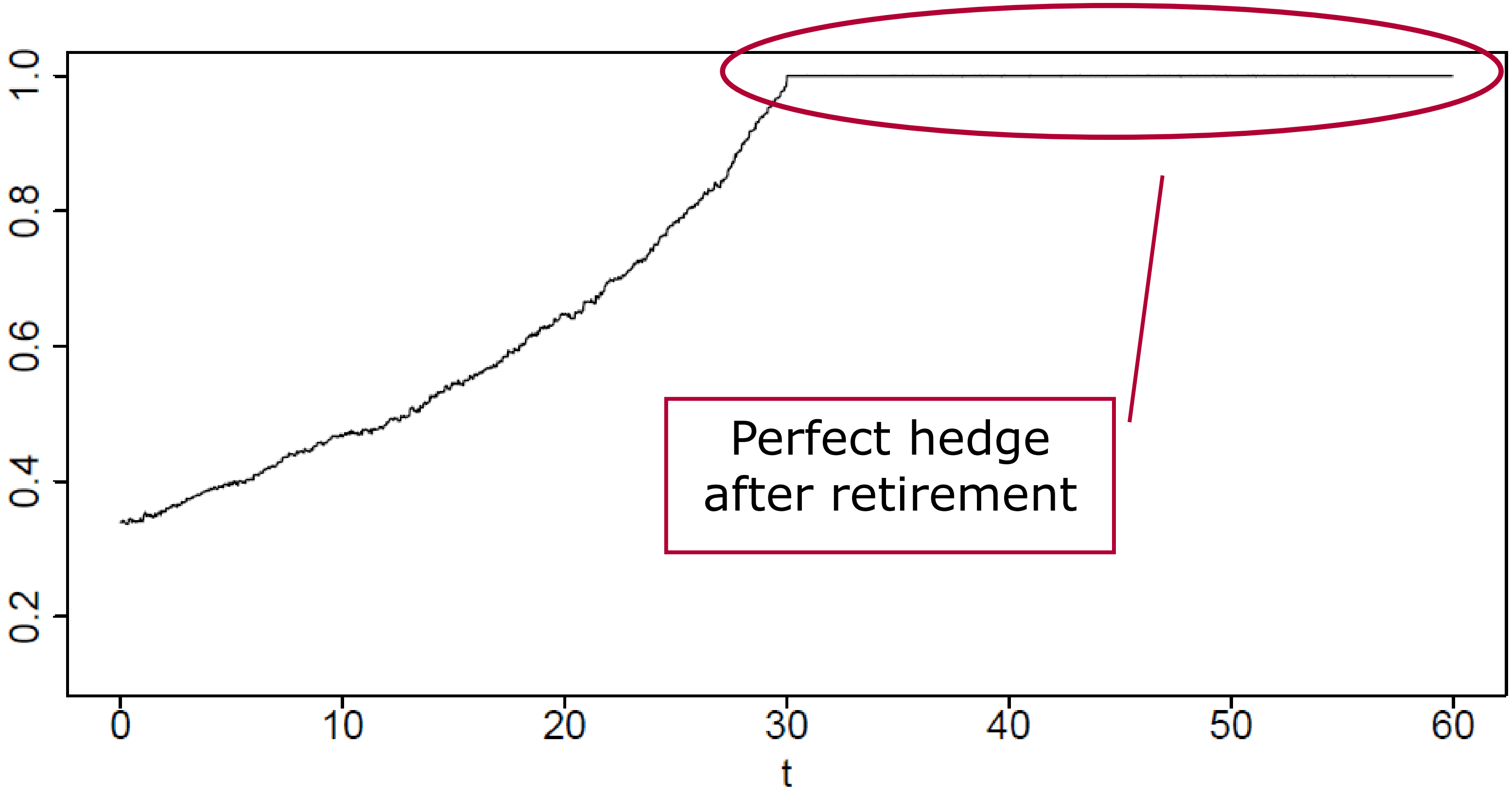
The hedging instruments
Intrinsic value processes for survivor swap on the insurance portfolio (left plot) and survivor swap on the population (right plot) in the two scenarios

SIMULATED INTRINSIC VALUE PROCESS



The liability - to be hedged!
Intrinsic value processes for the insurance contract in the two scenarios. Example: Age 30, Life annuity starting at age 60; yearly premiums n_1=100, n_2=1000,

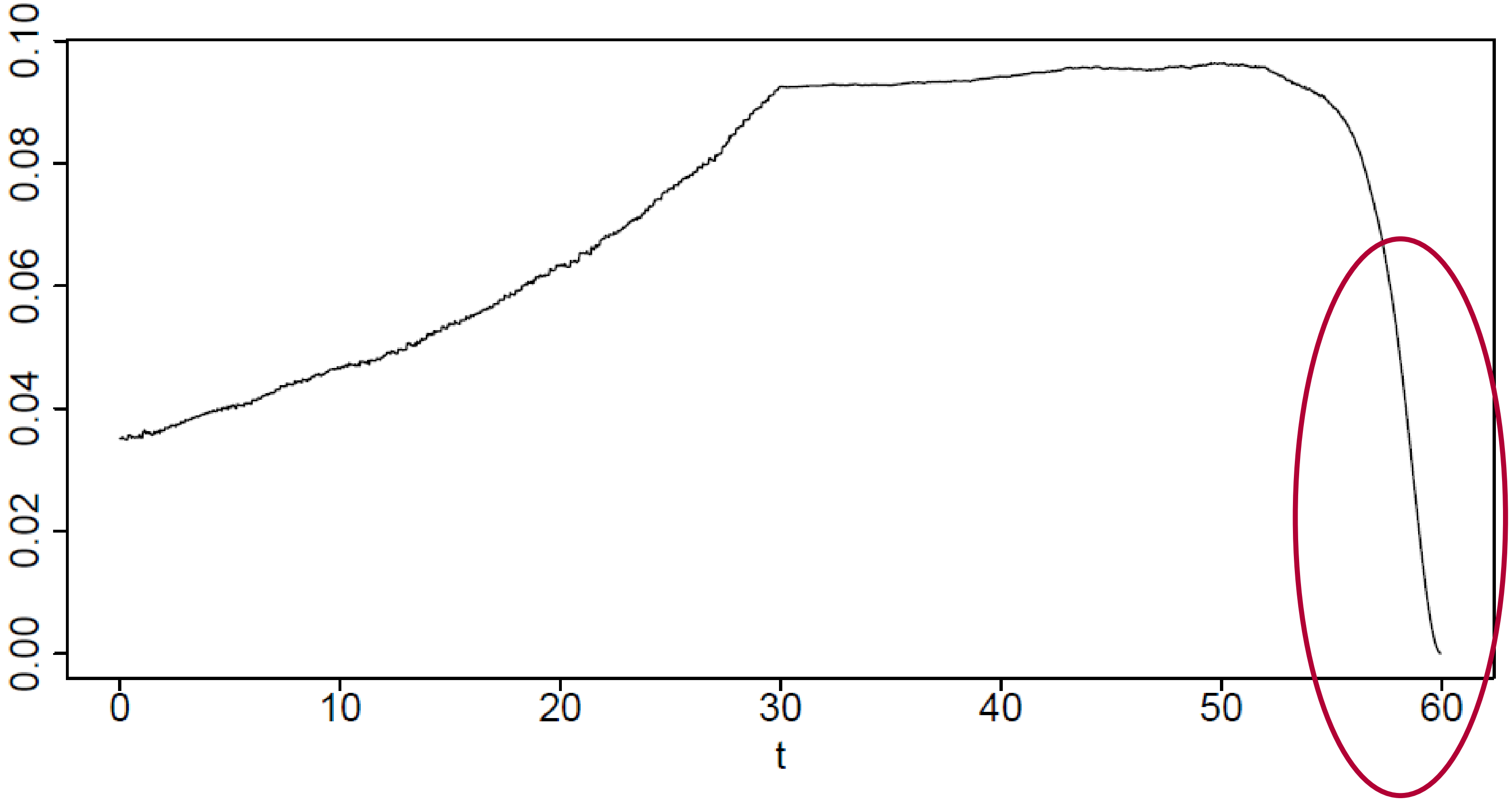
SIMULATED OPTIMAL SURVIVOR SWAPS (OWN PORTFOLIO)



Perfect hedge after retirement

Number of survivor swaps on the insurance portfolio held at time *t* in the market (B, P, Z_1)

SIMULATED OPTIMAL SURVIVOR SWAPS (POPULATION)

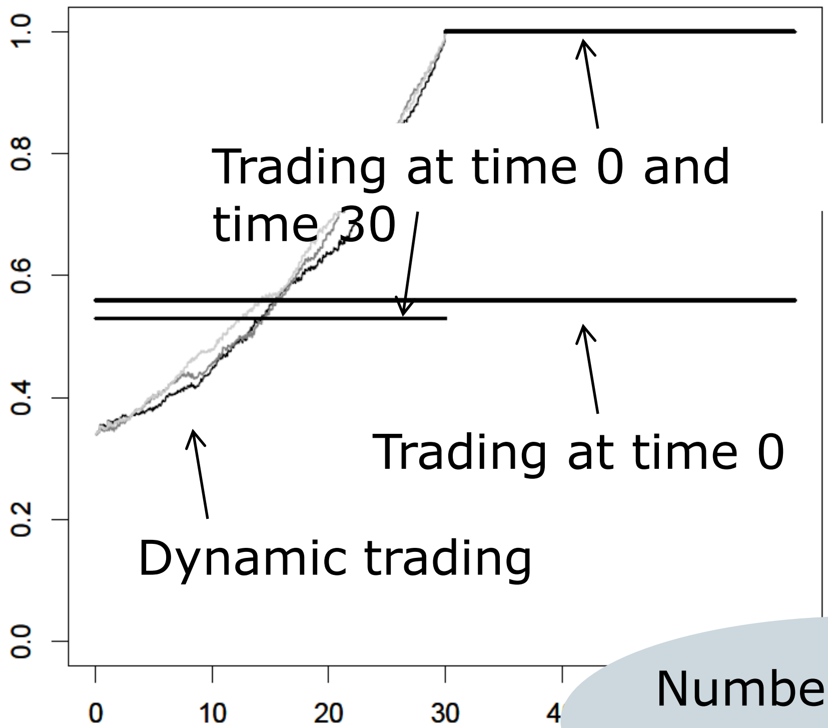


Number of survivor swaps on the population held at time *t* in the market (B, P, Z_2)

Swaps on population are not useful here due to short time horizon. Main risk is unsystematic risk

ILLIQUID SURVIVOR SWAPS: COMPARISON OF DYNAMIC AND STATIC TRADING STRATEGIES

Swaps on insurance portfolio

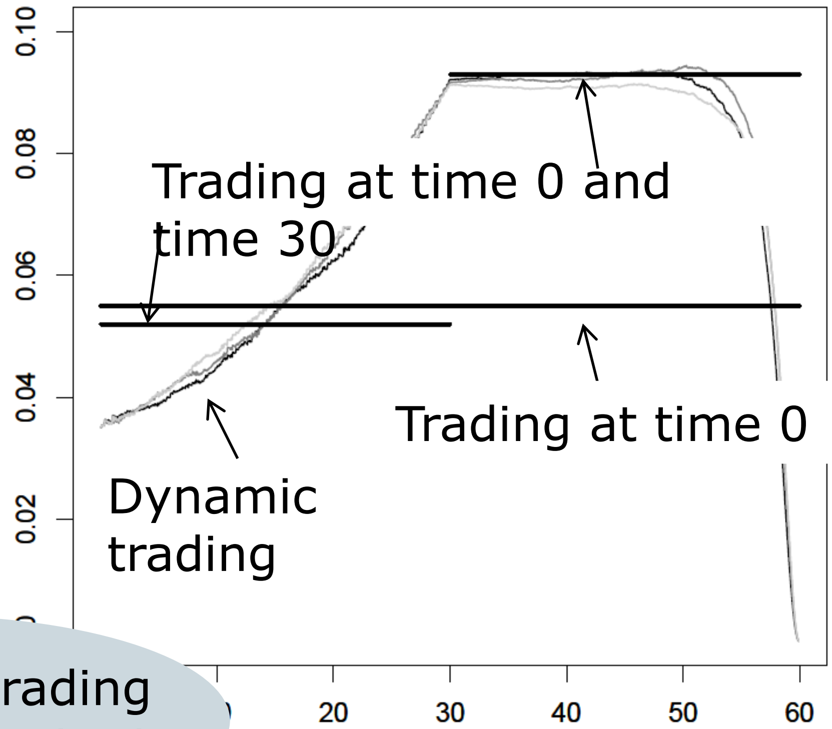


Trading at time 0 and time 30

Trading at time 0

Dynamic trading

Swaps on population portfolio



Trading at time 0 and time 30

Trading at time 0

Dynamic trading

Number of trading times is important

EFFICIENCY

Comparison of initial intrinsic risk

Dynamic trading insurance portfolio

Static hedging time 0 insurance portfolio

Static hedging time 0 and 30 insurance / population portfolio

n_1	n_2	No swap	D_1	$S_{1.1}$	$S_{1.2}$	$S_{2.2}$
100	**1,000**	**0.111**	**0.048**	**0.105**	**0.073**	**0.104**
100	**10,000**	**0.111**	**0.048**	**0.105**	**0.073**	**0.100**
1,000	**10,000**	**0.062**	**0.032**	**0.045**	**0.038**	**0.039**
1,000	**100,000**	**0.062**	**0.032**	**0.045**	**0.038**	**0.037**
10,000	**100,000**	**0.055**	**0.013**	**0.022**	**0.019**	**0.024**

Good results even with few trading times

CONCLUSIONS

Survivor swaps:

- Match sensitivity towards mortality and longevity risk
- Optimal number is a weighted assessment of systematic and unsystematic mortality risk
- Size of hedging portfolio matters
  - Small external hedging portfolios introduce new unsystematic risk and will not be efficient as hedging instrument
- Swap on own portfolio or on (other) total population:
  - Own portfolio most efficient – may be expensive. Less liquid
  - Other large reference portfolio can work very well. More liquid

Compared dynamic to mixed dynamic and static strategies

Intrinsic risk is almost unaffected by restriction to static strategies

RELATED RESEARCH ON MORTALITY MODELING AND MORTALITY RISK MANAGEMENT

HEDGING WITH SURVIVOR SWAPS

Dahl, Glar, Møller (2011). Mixed dynamic and static risk-Minimization with an application to survivor swaps, *European Actuarial Journal*

MORTALITY RISK AND MORTALITY DERIVATIVES

Dahl, Møller (2009). Mortality derivatives: Longevity Bonds and survivor swaps, *Finans/Invest* (In Danish)

DYNAMIC HEDGING WITH MORTALITY DERIVATIVES

Dahl, Melchior, Møller (2008). On systematic mortality risk and risk-minimization with survivor swaps, *Scandinavian Actuarial Journal*

VALUATION AND HEDGING WITH TRADITIONAL BONDS

Dahl, Møller (2006). On valuation and hedging of insurance contracts with systematic mortality risk, *Insurance: Mathematics and Economics*