Comparing Life Insurer Longevity Risk Transfer Strategies in a Multi-Period Valuation Framework

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Motivation

- Increased interest in reinsurance and longevity bonds to manage longevity risk for products that guarantee a retirement income (life annuities, pensions)

- Longevity risk management strategies
  - Capital and product pricing under different solvency regimes
    Nirmalendran et al. (2012)
  - Reinsurance (Olivieri, 2005; Olivieri and Pitacco, 2008; Levantesi and Menzietti, 2008)
  - Securitization (Cowley and Cummins, 2005; Wills and Sherris, 2010; Biffis and Blake, 2010; Gupta and Wang, 2011)

- Each strategy involves differing costs and risks

- Research Question: How do longevity risk management decisions impact the firm’s value for an insurer issuing life annuities allowing for frictional costs, market premiums, and solvency?
Investigate the impact of longevity risk transfer strategies on an insurer’s solvency and shareholder value for an annuity portfolio.

A multi-period valuation framework: one of the main contributions of the paper, allows for

- The costs of transferring longevity risk.
- Regulatory capital requirements and capital relief.
- Cost of holding capital.
- Financial distress costs.
- Policyholders’ price-default-demand elasticity.

Analyze the interaction between capital management and reinsurance or securitization.

Valuation approaches

- Economic Balance Sheet (EBS)
- Market-Consistent Embedded Value (MCEV)
Introduction

- Stochastic mortality model with both systematic and idiosyncratic longevity risk.

Risk transfer strategies
- Reinsurance: indemnity-based, covers both systematic and idiosyncratic longevity risk.
- Securitization: index-based, covers only systematic longevity risk.

Solvency capital requirements - Solvency II.

Results: Longevity risk management strategies...
- reduce the insurer’s default probability.
- increase shareholder value and,
- reduce the volatility of the shareholder value.
- reduce the level and the volatility of frictional costs.
- reduce investor uncertainty.
Affine Mortality Model

- Stochastic mortality model by Blackburn and Sherris (2012).
  - Based on forward (cohort) mortality rates
  - Avoids need for nested simulations at future time points when valuing future liabilities.
  - Model structure: HJM forward rate models (Heath et al., 1992).
  - Model gives stochastic forward interest rates and forward mortality rates.

- Use a model variant with 2-stochastic mortality risk factors, a deterministic volatility function and Gaussian dynamics (Blackburn, 2013).

- Model is calibrated to Australian male population ages 50-100, years 1965-2007
Pricing Measure

- Risk-neutral measure: best estimate cohort survivor curve, used to value annuity cash flows without loading.
- Pricing and market valuation measure: construct a new martingale measure.
  - \( \lambda \): constant price of risk: instantaneous Sharpe ratio (Milevsky and Promislow, 2001).
  - No impact on the volatility function, but scaling of the initial forward mortality curve.
  - Calibrate from quoted reinsurance loadings (survivor swap premium): \( \lambda = 0.1555 \)

- Assume interest and mortality rates are independent.
- Assume interest rates are deterministic.
Pricing Measure

- Best estimate and market pricing survivor curves with 99% confidence intervals.

*Figure: Cohort Survival Distribution Aged 65 in 2010*
Framework

- Monte-Carlo simulation of an insurer with an annuity portfolio
  - Portfolio run-off from ages 65 to 100
  - Annuity demand related to premium loading and default probability

- Idiosyncratic risk due to portfolio size

- Risk transfer (static hedge) through:
  - Survivor Swap - indemnity based
  - Survivor Bond - index based

- Risk transfer options
  - 50% or 100% risk transfer
  - 50% or 100% capital relief

- Mark-to-Market valuation of liabilities / reserve

- EBS and MCEV balance sheet items
  - Frictional costs due to holding capital
  - Recapitalization costs
  - Excess capital distributed as dividends
  - Initial shareholder capital
  - Expenses
Monte Carlo Simulation and Idiosyncratic Longevity Risk

- Implement the mortality model as a discrete time version of the HJM model.
- Use Monte Carlo simulation based on Glasserman (2003).
- For each simulation path $m$:
  - Generate mortality rates to give a survivor index.
  - Generate forward mortality curves for each discrete time point $t_i$.
  - Expected number of survivors: $\hat{I}(m)(t_i; x)$
- Idiosyncratic longevity risk:
  - Random death times for individuals: the first time the implied force of mortality for path $m$ is above $\varrho$.
  - $\varrho$ is an exponential random variable with parameter 1.
  - Gives the actual number of survivors: $\tilde{I}(m)(t_i; x)$
Idiosyncratic Longevity Risk

Figure: Portfolio Survivors $\tilde{I}^{(m)}(t; x)$
Annuity Pricing and Reserving

- The **market value** of an annuity that pays $b$ per year to each annuitant in a cohort age $x$ at time-0 is

$$
\tilde{a}(0, t_n; x) = \sum_{t_s=t_1}^{t_n} b \cdot \exp \left( - \sum_{t_j=t_0}^{t_{s-1}} \left( f(0, t_j) + \tilde{\mu}(0, t_j; x) \cdot [t_{j+1} - t_j] \right) \right).
$$

- The path dependent **forward market value** of an annuity is

$$
\tilde{a}(0, t_i, t_n; x) = \sum_{t_s=t_{i+1}}^{t_n} b \cdot \exp \left( - \sum_{t_j=t_0}^{t_{s-1}} \left( f(0, t_j) + \tilde{\mu}(0, t_j; x) \cdot [t_{j+1} - t_j] \right) \right).
$$

- The **fair value** of an annuity that pays $b$ per year to each annuitant in a cohort age $x$ at time-0 is

$$
\tilde{a}(0, t_n; x) = \sum_{t_s=t_1}^{t_n} b \cdot \exp \left( - \sum_{t_j=t_0}^{t_{s-1}} \left( f(0, t_j) + \tilde{\mu}(0, t_j; x) \right) \cdot [t_{j+1} - t_j] \right),
$$

where \( \tilde{\mu}(0, t_j; x) \) is the best estimate cohort forward survivor curve.
Portfolio

- Annuity single premium, $\gamma^P$ - premium loading

$$\pi = b \cdot \left(1 + \gamma^P\right) \cdot \tilde{a}(0, t_n; x).$$

- Market reserve - unhedged

$$\tilde{V}_p^{(m)}(t_i; x) = \tilde{l}^{(m)}(t_i; x) \cdot \tilde{a}^{(m)}(t_i; x).$$

- Market reserve - hedged

$$\tilde{V}_h(t_i; x) = n_0 \cdot \tilde{S}(0, t_i; x) \cdot \tilde{a}(0, t_i, t_n; x).$$

- Total portfolio reserve

$$\tilde{V}_s^{(m)}(t_i; x) = (1 - \omega_h)\tilde{V}_p^{(m)}(t_i; x) + \omega_h \tilde{V}_h(t_i; x).$$
Portfolio

- Solvency Capital Requirement - $\phi = 0.2$

\[ \tilde{M}_p^{(m)}(t_i) = \tilde{V}_p^{(m)}(t_i)| \text{Longevity shock} - \tilde{V}_p^{(m)}(t_i) \]

- Total SCR, assuming $\omega_c$, is the proportion of hedged liabilities that are given capital relief.

\[ \tilde{M}_h^{(m)}(t_i) = \tilde{M}_p^{(m)}(t_i) \cdot (1 - \omega_c). \]

- Total Reserve

\[ \tilde{V}^{(m)}(t_i) = \tilde{V}_s^{(m)}(t_i) + \tilde{M}^{(m)}(t_i) + \tilde{V}_e^{(m)}(t_i) \quad (1) \]
Cash Flows

- No hedging

\[
\widetilde{CF}^{(m)}(t_i) = -b \cdot \widetilde{l}^{(m)}(t_i; x) - \widetilde{E}^{(m)}(t_i).
\]

- Survivor Swap

\[
= -b \cdot \left[\widetilde{l}^{(m)}(t_i; x) + \omega_h \left(1 + \gamma^R \right) \cdot \widetilde{S}(0, t_i; x) - \widetilde{l}^{(m)}(t_i; x)\right] - \widetilde{E}^{(m)}(t_i).
\]

- Survivor Bond

\[
= -b \cdot \left[\widetilde{l}^{(m)}(t_i; x) + \omega_h \left(1 + \gamma^R \right) \cdot \widetilde{S}(0, t_i; x) - \widetilde{l}^{(m)}(t_i; x)\right] - \widetilde{E}^{(m)}(t_i).
\]
Solvency, Dividends, and Recapitalization

- $\tilde{A}^{(m)}(t_i) < \tilde{V}_s^{(m)}(t_i)$: there are insufficient assets to cover time-$t$ liabilities and the insurer defaults.
  - Annuitants receive only the residual assets
  - Limited Liability Put Option:
    $$\text{LLPO}^{(m)}(t_i) = \max\{0, \tilde{V}_s^{(m)}(t_i) - \tilde{A}^{(m)}(t_i)\}.$$

- $\tilde{A}^{(m)}(t_i) - \tilde{V}^{(m)}(t_i) < 0$: no default, but insufficient capital to meet regulatory obligations. The shortfall, $\tilde{R}^{(m)}(t_i)$, is recapitalized from shareholders.

- $\tilde{A}^{(m)}(t_i) - \tilde{V}^{(m)}(t_i) \geq 0$: no default and enough capital to meet regulatory requirements. The excess capital is distributed to shareholders as a dividend, $\tilde{D}^{(m)}(t_i)$. 
Annuity Demand

- Exponential demand function (Zimmer et al., 2009, 2011)
  - Default sensitivity $\alpha$, price sensitivity $\beta$
  - Annuity premium loading factor $\gamma^P$
  - Cumulative default probability $d$

$$\phi^*(\gamma^P, d) = e^{(\alpha \cdot d + \beta \cdot \gamma^P + \theta)}.$$ 

- The number $n_0$ of annuities sold at time-0
  - $n_m$ is the total market size

$$n_0 = n_m \cdot \phi^*(\gamma^P, d).$$
Economic Balance Sheet (EBS)

- Frictional Costs: \( \tilde{FC}^{(m)}(t) = \rho \cdot [\tilde{V}^{(m)}(t) - \tilde{V}_s^{(m)}(t)] \)
- Recapitalization Costs: \( \tilde{FC}_R^{(m)}(t) = \psi \cdot \tilde{R}^{(m)}(t) \)
- LLPO: see slide 16
- \( X(0) \) represents shareholder value at time-0

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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</thead>
<tbody>
<tr>
<td>( \Pi )</td>
<td>( V_s^{(m)}(0) )</td>
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<td></td>
<td>( \tilde{PV}_{FC}^{(m)}(0) )</td>
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<td></td>
<td>( \tilde{PV}_{E}^{(m)}(0) )</td>
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<td>( -LLPO(0) )</td>
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<td>( X(0) )</td>
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Market-Consistent Embedded Value (MCEV)

- The present value of future profits

\[
\widetilde{FP}^{(m)}(t_i) = \sum_{t_s = t_{i+1}}^{t_{n-1}} \left[ \left( \widetilde{V}^{(m)}(t_s) - \widetilde{V}^{(m)}(t_{s-1}) \right) + i \cdot \widetilde{A}^{(m)}(t_{s-1}) + \widetilde{CF}^{(m)}(t_s) \right] \cdot \nu(t_i, t_s).
\]

- The Value of the In-Force business (VIF)

\[
VIF(t) = \widetilde{FP}^{(m)}(t_i) - \widetilde{PV}_{FC}^{(m)}(t_i) - \widetilde{PV}_{FCR}^{(m)}(t_i) + \widetilde{LLPO}(t_i).
\]

- MCEV at time-\(t_i\) is

\[
MCEV(t_i) = VIF(t_i) + E^Q(t_i), \tag{2}
\]

- where \(E^Q(t_i)\) is the time-\(t_i\) equity of the insurer.
- Valuation at \(t = 0\): \(E^Q(t_i) = 0\), SHV is \(VIF(t_i)\).
Results

- Assume fixed one-year default probability of 0.5%.
- Portfolio size depends on premium loading.
- Insurer’s actual default probability depends on premium loading.

(a) Demand function: portfolio size.

(b) Insurer's actual default probability.
**Results: One-Year Default Probabilities**

- Bond: higher premium loading - smaller portfolio size - higher default prob.
- Swap: not a problem, idiosyncratic risk is hedged
- No effect of capital relief when insurer is fully hedged.

![Graph of default probabilities for bonds and swaps](image-url)
Results: Shareholder Value

- Longevity risk transfer: small gains to the expected VIF and EV values for any fixed premium loading.
- Reason: reduction of frictional costs.
Results: Volatility of Shareholder Value

- Longevity risk transfer reduces the volatility of SHV
- Here: SHV from the Economic Balance Sheet

(g) X (Std) Bond
(h) X (Std) Swap
Results: Volatility of Shareholder Value

- Longevity risk transfer reduces the volatility of SHV
- Here: \textbf{MCEV} (=VIF in our model)

\begin{itemize}
  \item \textbf{MCEV (Std) Bond}
  \item \textbf{MCEV (Std) Swap}
\end{itemize}
Results: Frictional Costs

- $\text{FC: } \widetilde{FC}^{(m)}(t) = \rho \cdot [\widetilde{V}^{(m)}(t) - \widetilde{V}_s^{(m)}(t)]$

- Longevity risk transfer reduces the expected value and the volatility of frictional costs.

\begin{align*}
\text{(k) Frictional Costs} \\
\text{(l) Frictional Costs (std)}
\end{align*}
Results: Financial Distress Costs

- Longevity risk transfer reduces the expected value and the volatility of recapitalization costs.

(m) Financial Distress Costs

(n) Financial Distress Costs (std)
Conclusion

- Stochastic mortality model with systematic and idiosyncratic longevity risk
- Test risk transfer strategies
  - Survivor Swap: indemnity based
  - Survivor Bond: index based
- EBS and MCEV valuation methods
- Maintain Solvency II SCR
- Benefits of Longevity risk transfer
  - Reduce the insurer’s default probability.
  - Increases shareholder value and,
  - Reduces the volatility of the shareholder value.
  - Reduces the friction costs and the volatility of friction costs.
  - Reduces the volatility of dividend payment and recapitalization requirements.
References


